

# Trends, Cycles and Lost Decades

## Decomposition from a DSGE Model

### with Endogenous Growth\*

Ryo Hasumi<sup>†</sup> Hirokuni Iiboshi<sup>‡</sup> Daisuke Nakamura<sup>§</sup>

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#### Abstract

In this paper we incorporate endogenous productivity growth into a medium-scale new Keynesian dynamic stochastic general equilibrium (DSGE) model, to which a new shock regarding R&D activities is added. By matching the model parameters to the Japanese economy from 1980:Q2 to 2013:Q4 and decomposing the output into trend and cycle components, we find that the stagnation of the so-called lost decades was caused by a decline in economic growth as well as major recessions in the business cycle. The common trend estimated by our model is based on multiple time series data and is much more volatile than the trend extracted by either the Hodrick-Prescott or the band-pass filter.

*Keywords:* endogenous TFP growth, New Keynesian DSGE, trend shift, technological change

## 1 Introduction

After the bubble economy had reached the point of collapse in January 1991, the Japanese economy experienced a long period of stagnation, the so-called “lost decades.” During this period, two other economic crises occurred, specifically the Asian currency crisis in 1997 and the collapse of Lehman Brothers in 2008. These repeated crises are likely to have made the growth rate of the real GDP reverse from upwards to downwards. The

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<sup>†</sup>Japan Center for Economic Research

<sup>‡</sup>Graduate School of Social Sciences, Tokyo Metropolitan University

<sup>§</sup>Consumer Affairs Agency, Government of Japan.

sources and magnitudes of the stagnation during the lost decades are analyzed by many economists and policy makers. However, the topic seems to be controversial, and we can obtain no consensus for this debate. For instance, Hayashi and Prescott (2002) points out that the slowdown of total factor productivity (TFP) must be a key factor in the long stagnation. This is because the stagnation should be regarded not as a decline caused by temporary shocks mainly triggered by the demand side but as permanent shocks of production. If so, it is plausible that there was a huge decline on the supply side but no change on the demand side behind this severe situation. In addition, Kaihatsu and Kurozumi (2014) estimates the extent to which the financial accelerator of the banking sector influenced the decline of the lost decades, but financial friction contributed a tiny portion to it in Japan.

On the other hand, central banks and policy organizations in developed countries have aggressively adopted the dynamic stochastic general equilibrium model based on the new Keynesian (NK) framework to analyze business cycles and the effects of monetary and fiscal policies on macroeconomics, particularly since the millennium. Earlier empirical studies focusing on Japan, such as those by Sugo and Ueda (2008), Kaihatsu and Kurozumi (2014) and Iiboshi, Matsumae, Namba and Nishiyama (2015), determine that the price and wage rigidities and habit formation of consumption assumed in NK models are important factors to grasp comovements among economic series representing business cycles as well as the variation in TFP. However, these models are set up to focus on short-term rather than long-term frequency. Comin and Gertler (2006) open a crack in this drawback. They incorporate the endogenous growth model proposed by Romer (1990) into a real business cycle (RBC) model and decompose the output series between business cycles and trends in terms of the macroeconomic framework. Thanks to their contribution, we can concentrate simultaneously on both sides – the short-term and long-term co-movements – of various macroeconomic series. For example, Guerron-Quintana and Jinnai (2015) and Guerron-Quintana, Hirano and Jinnai (2016) analyze financial crises' impact on the growth rate of the Great Recession for the US economy using the framework of Comin and Gertler (2006).

In this paper we extend the RBC model by Comin and Gertler (2006) to the NK framework and empirically analyze the long stagnation in Japan by estimating the extended model. In this sense we follow Ikeda and Kurozumi (2014), who expand Comin and Gertler's (2006) model by embedding price and wage rigidities and financial friction. We incorporate the persistence of habit consumption and index rules of prices and wages into Ikeda and Kurozumi (2014)'s model. Using this model, we extract the trend component from the real GDP and classify the factors contributing to the growth from the historical decomposition for Japan after 1981:Q2. Since the historical decomposition would show the exogenous shocks to which the decline in the output in the three

economic crises is attributed, we reconsider the sources of the lost decades from the viewpoint of macroeconomics.

The contributions of our paper are as follows. (1) We estimate a medium-scale NK model, which adopts nominal rigidities of prices and wages and habit formation of consumption and focuses on short-term fluctuations, to which the R&D endogenous growth framework by Comin and Gertler (2006) is added. Our model adopts both sides of short-run and long-run fluctuations by combining business cycle and growth models. In addition, we conduct model selection between our model and a standard NK model without the endogenous growth framework. (2) By embedding a new shock regarding R&D activities and using estimated parameters, we empirically classify the factors that are attributable to the long stagnation. Specifically, we calculate the historical and variance decompositions of the common trend and business cycle components. (3) This paper is the first empirical attempt to apply Comin and Gertler's (2006) model to Japan.

Our empirical findings are follows. First, from the model selection in terms of the marginal likelihood of Japan's recent data set, a standard NK model without the endogenous growth model is superior to our model. However, the assumption of nominal rigidities of prices and wages is likely to work very well even for a long period, for example over three decades, since the Calvo price and wage parameters are high values and this model dominates its counterpart without the rigidities. Second, the R&D activity and investment shocks account for the majority of the business cycle components of the real GDP and investment. However, the R&D shock affects investment and consumption in opposite directions, while the investment shock affects them in the same direction. Third, our estimated common trend fluctuates with considerable volatility, similar to the trend of investment extracted by the Hodrick-Prescott (HP) and band-pass (BP) filters. In contrast, the trend components of output and consumption of both the HP and the BP filter have little volatility. Fourth, two deep declines in the growth rate during the Asian financial crisis in 1998 and the Lehman Brothers' failure in 2008:Q3 caused not only major recessions in the business cycles but also stagnation of the economic growth.

The rest of our paper is organized as follows. Section 2 describes the motivation of this study. Our model and estimation method are explained in Sections 3 and 4, respectively. Section 5 deals with the estimation results. We conclude in Section 6. In the Appendix we show that the equilibrium conditions consist of the first-order conditions (FOCs) and restrictions.

## 2 Motivation

### Growth Decline in the “Lost Decades”

Hayashi and Prescott (2002) emphasize that the huge decline of economic growth at the beginning of the 1990s resulted from a substantial reduction of total factor productivity (TFP). On the other hand, other empirical studies support the idea that the long stagnation in this period occurred due to the reluctance of financial institutions to finance the corporate sector. Besides, R&D must be an important factor in realizing and sustaining strong economic growth. This paper focuses on the effect of TFP on growth along the line of Hayashi and Prescott (2002) and attempts to decompose the time series of the TFP into several aspects based on DSGE and growth models. The decomposition might provide a clue for identifying the sources of the “lost decades.”

### Common Stochastic Trend and Endogenous Growth

There is a large literature considering the theoretical and empirical aspects of non-stationary univariate and multivariate time series, including the random-walk process and co-integration. Many empirical studies report that most macroeconomic series, such as GDP, consumption and investment, follow a non-stationary process. Furthermore, other empirical studies estimate the decomposition between the cycle component and the trend component, since macroeconomic series are thought to consist of a stationary process regarded as business cycles and a non-stationary process indicating a stochastic trend or economic growth.

King, Plosser, Stock and Watson (1991) examines the long-run relationship made by permanent productivity shocks, regarded as shocks to the common stochastic trend of output, consumption and investment based on RBC theory, using a co-integration test. Altig, Christiano, Eichenbaum and Linde (2011) expands their research to a DSGE model to express a model-based co-integration system by including an investment-specific shock with a random-walk process. Most of the latest DSGE models, for instance those by Adolfson, Laséen, Lindé and Villani (2007) and Christiano, Trabandt and Walentin (2011), follow their unit root technology shock, inducing the common stochastic trend.

This study also adopts their theory of the growth rate and expands it by incorporating an endogenous growth model. Generally speaking, primary macroeconomic indexes, such as output, consumption and investment per capita and real wage, are considered to include the common stochastic trend,  $\log A_t$ , which makes a long-run stable relationship among them, that is,

$$\log(Y_t/N_t) = \tilde{y}_t + \log A_t,$$

$$\log(C_t/N_t) = \tilde{c}_t + \log A_t,$$

$$\log(I_t/N_t) = \tilde{i}_t + \log A_t,$$

$$\log W_t = \tilde{w}_t + \log A_t,$$

where  $\tilde{y}_t$ ,  $\tilde{c}_t$ ,  $\tilde{i}_t$  and  $\tilde{w}_t$  are idiosyncratic stationary time series around the steady state indicating business cycle components. According to Altig et al. (2011), the common stochastic trend,  $\log(A_t)$ , follows a non-stationary autonomous process with  $I(1)$  and the first difference of the trend is a stationary process,

$$\log \mu_t^A = \rho \log \mu_{t-1}^A + z_t, \text{ for } -1 < \rho < 1$$

where  $\log \mu_t^A$  is  $\log \mu_t^A = \log A_t - \log A_{t-1}$ , and  $z_t$  is an exogenous technology shock. On the other hand, the trend of our model follows an endogenous process:

$$\log A_t = \log A_{t-1} + \frac{\lambda_x}{1 - \alpha} \log \mu_t^V,$$

where  $\mu_t^V$  is the growth rate of the goods variety given as

$$\mu_t^V = (1 - \delta_a) + \Delta_{a,t}.$$

The constant term on the right hand side of the equation above,  $1 - \delta_a$ , represents the deterministic growth rate, and stochastic growth is generated from the dynamics of an endogenous variable,  $\Delta_{a,t}$ , defined as the additional goods variety.

The New Keynesian model mainly focuses on the contemporaneous cross-sectional relation of the business cycle components, whereas the endogenous growth model focuses on the trend component. Our study evaluates aspects using the above strategy.

#### **Four Potential Sources of the “Lost Decades”**

In the following sections, we estimate the cause of the decline in economic growth during the “lost decades” through the lens of a DSGE model with endogenous growth. As described above, the shift in the technology growth rate,  $\mu_t^A$ , is determined by the growth rate of the goods variety,  $\mu_t^V$ , whereas the additional goods variety,  $\Delta_{a,t}$ , implemented by R&D activity, is affected by structural shocks from economic activity and market clearing. Hence, by measuring the contribution of structural shocks to the additional goods variety, we try to identify the extent to which the sources account for the decline in

growth. The shocks considered in our study are classified into the following four aspects:

- The effect of TFP and physical investment
- The effect of R&D investment
- The effect of consumption
- The effect of monetary and fiscal policies

We exclude the financial sector from our model, since it can be assumed to have an indirect effect on economic growth via the above four paths. Accordingly, our study just focuses on the four direct effects on economic growth.

### 3 Model

Our model adopts Comin and Gertler (2006)'s endogenous growth framework by R&D investment. Ikeda and Kurozumi (2014) expands it by embedding price and wage rigidities and financial friction, and we follow this strategy. The novelty of our model is an R&D success probability shock embedded in the R&D sector, which represents the efficiency of a firm's R&D activities. Furthermore, the model incorporates consumption habits and index rules of prices and wages to improve the fitness of the consumption, price and wage to the data.

In the model the economy consists of three sectors, namely R&D, producers and households, with seven agents. The R&D sector contributes to economic growth by developing innovative technology, bringing new products manufactured by the productive sector. In this framework innovation increases the variety of intermediate goods to hold symmetry of the two relations among different goods firms, that is, (1) retail goods vs. wholesale goods and (2) final goods vs. intermediate goods, as described below.

#### 3.1 R&D Sector

##### Innovator

The innovator is assumed to be a representative agent who creates new innovation,  $I_{d,t}$ , which is useful for increasing the quantity of intermediate goods,  $X_{f,t}(h)$ . To achieve this, he uses retail goods,  $Y_t$ . Then he sells the right to his innovation to an adopter, who converts the innovation into newly developed final goods via intermediate goods.

Let  $Z_t$  be his total stock of innovation and  $\delta_z$  the obsolescence rate of the stock. Then we obtain this dynamics as

$$Z_t = (1 - \delta_z)Z_{t-1} + \Phi_t I_{d,t}, \tag{1}$$

where  $\Phi_t$  is the R&D productive parameter that transforms additional innovation into stock.  $\Phi_t$  is given as

$$\Phi_t \equiv \chi_z \left( \frac{Z_{t-1}}{A_{t-1}} \right)^\rho \left( \frac{Z_{t-1}}{I_{d,t}} \right)^{1-\rho}, \quad (2)$$

with  $0 < \rho \leq 1$  and where  $\chi_z > 0$  is a scale parameter.  $A_t$  represents the level of technology, and the technology progress rate of our model,  $\mu_t^A$ , can be written as the logarithm of its ratio:  $\log(A_t/A_{t-1})$ . Eq.(2) shows the congestion effect of the innovation stock in which a larger  $A_{t-1}$  than the steady state reduces the value of  $\Phi_t$ . Since the innovator faces perfect competition, he optimizes his profit and gains zero profit, satisfying the no-arbitrage condition, such as

$$1 = \Phi_t(1 - \delta_z) \mathbf{E}_t \Lambda_{t|t+1} J_{t+1}, \quad (3)$$

where  $\mathbf{E}_t \Lambda_{t|t+1}$  is the stochastic discount factor (SDF) of households and  $J_t$  is the value of the innovation described in the following part. Eq. (3) indicates the equivalent exchange between the innovation and the retail goods, of which the price is unity. From Eq.(1) and Eq.(3) we obtain the dynamics of the innovator as

$$I_{d,t} = (1 - \delta_z) \{Z_t - (1 - \delta_z)Z_{t-1}\} \mathbf{E}_t \Lambda_{t|t+1} J_{t+1}. \quad (4)$$

## Adopter

The adopter is categorized as a representative agent who converts the available technology acquired from the innovator into a new product of intermediate goods. To buy the right to the innovation, he obtains loans from households, and he tries to manufacture a new product using the retail goods,  $Y_t$ . If he is successful, he sells it to intermediate goods producers.

The value of a premature product, which has not yet been adopted, to the adopter is obtained as

$$J_t = \max_{I_{a,t}} [-I_{a,t} + (1 - \delta_a) \{ \lambda_t P_t^V + (1 - \lambda_t) \mathbf{E}_t \Lambda_{t|t+1} J_{t+1} \}], \quad (5)$$

where  $I_{a,t}$  is the cost of investment for adoption and  $\delta_a$  is the obsolescence rate of the adopted technology.  $\lambda_t$  is the success probability of converting the innovation into a new product.  $P_t^V$  denotes the value of the adopter successfully developing a new product, which indicates the present value of profit of the adopter. The success probability is an endogenous variable given as

$$\lambda_t \equiv \lambda_0 \left( \frac{V_{t-1}}{A_{t-1}} I_{a,t} \right)^{\omega_a} z_t^\lambda, \quad (6)$$

where  $\lambda_0 > 0$ ,  $\omega_a > 0$ ,  $V_t$  denotes the stock of adopted innovation, or the variety of final goods, and  $z_t^\lambda$  is an autoregressive (AR) process of a structural shock regarded as an *R&D success probability shock*. This shock represents the efficiency of the adopter's R&D activities.

Since there is a spillover effect of technology, we assume that it has a certain degree of inertia<sup>1</sup>. Eq.(6) also shows the congestion effect of adoption slowing down the speed of accumulation of  $V_t$ . The link between the level of technology and the stock of adopted innovation  $V_t$  can be expressed as

$$A_t = V_t^{\frac{\lambda_x}{1-\alpha}}, \quad (7)$$

where  $\lambda_x$  is the markup rate of the price of final goods  $X_t$ , as described below. The increment,  $\Delta_{a,t}$ , of the adopted technology  $V_t$  is given as

$$\Delta_{a,t} \equiv (1 - \delta_a)\lambda_t(Z_{t-1} - V_{t-1}), \quad (8)$$

where the term in brackets is the stock of innovation that the adopter owns but has not yet adopted, and the first-order condition for investment,  $I_{a,t}$ , by maximizing Eq. (5) subject to Eq.(6) and Eq.(8), is written as

$$I_{a,t} = \omega_a(1 - \delta_a) (\lambda_t P_t^V - \lambda_t \mathbf{E}_t A_{t|t+1} J_{t+1}). \quad (9)$$

From Eq.(5) and Eq.(9) we obtain the value of unadopted innovation as

$$J_t = (1 - \delta_a) [(1 - \omega_a) \lambda_t P_t^V + \{1 - (1 - \omega_a) \lambda_t\} \mathbf{E}_t A_{t|t+1} J_{t+1}], \quad (10)$$

where  $J_t$  is also used as the optimization of the innovator as in Eq.(3).

### 3.2 Productive Sector

Firms are divided into four groups – (i) retailers, (ii) wholesalers, (iii) final goods firms and (iv) intermediate goods firms – based on the categories of goods generated in the production process. The wholesaler and intermediate goods firms face a specialized market under monopolistic competition, but only the former follow a Calvo-style price setting. The rest of the firms produce under perfect competition.

#### Relation among the Four Types of Goods

Before describing the producing agents, we show the relation among the four types of goods as follows.

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<sup>1</sup>The R&D success probability shock in our paper corresponds to the “barriers to technology adoption” introduced by Ikeda and Morita (2016).



- a). The retail goods  $Y_t$  are produced by bundling the wholesale goods  $Y_t(w)$  indexed by  $w \in [0, 1]$ , based on the CES production function, as

$$Y_t = \left( \int_0^1 Y_t(w)^{\frac{1}{1+\lambda_p}} dw \right)^{1+\lambda_p}, \quad (11)$$

where  $\lambda_p > 0$  denotes the net markup rate of the wholesale goods.

- b). The wholesale goods  $Y_t(w)$  are produced from the final goods  $X_{f,t}(w)$  indexed by  $f \in [0, V_{t-1}]$  and  $w \in [0, 1]$  with equal amounts as

$$Y_t(w) = X_{f,t}. \quad (12)$$

- c). The final goods  $X_{f,t}$  are produced by bundling the intermediate goods  $X_{f,t}(h)$  indexed by  $h \in [0, 1]$ , based on the CES production function, as

$$X_{f,t} = \left( \int_0^1 X_{f,t}(h)^{\frac{1}{1+\lambda_x}} dh \right)^{1+\lambda_x}, \quad (13)$$

where  $\lambda_x > 0$  is the net markup rate of the intermediate goods.

## Retailer

The retailer is a representative agent who produces retail goods  $Y_t$  by bundling a set of wholesale goods  $Y_t(w)$  indexed by  $w \in [0, 1]$ . Then the retail goods are sold to households, intermediate goods firms, innovators and adopters. Under the constraint of the CES production function (11), the retailer maximizes his profit, given as

$$\max_{\{Y_t(w)\}} P_t Y_t - \int_0^1 P_t(w) Y_t(w) dw, \quad (14)$$

where  $P_t$  and  $P_t(w)$  denote the prices of the retail goods and the wholesale goods, respectively. The FOC of the retailer's profit-maximizing problem indicates the demand function of the wholesale goods, given as

$$Y_t(w) = \left( \frac{P_t(w)}{P_t} \right)^{-\frac{\lambda_p}{1+\lambda_p}} Y_t, \quad (15)$$

## Wholesaler

There is a continuum of wholesalers, indexed by  $w \in [0, 1]$ . Each type of wholesaler produces its goods by using the final goods as its input. Under the constraint of the CES

production function (12), the wholesalers minimize their costs, written as

$$\min_{\{X_{f,t}\}} \int_0^{V_{t-1}} \frac{P_{f,t}^x}{P_t} X_{f,t} df - MC_t(w) Y_t(w) \quad (16)$$

where  $P_{f,t}^x$  denotes the price of the final goods and  $MC_t(w)$  means the marginal cost of producing wholesale goods, respectively. The FOC of the wholesaler's cost-minimizing problem suggests the marginal cost function of the wholesale goods, given as

$$MC_t(w) = \frac{P_{f,t}^x}{P_t}. \quad (17)$$

$MC_t(w)$  does not depend on the indexation variable  $w$ , so the marginal cost  $MC_t$  is common to each wholesaler. Hereafter, we denote  $MC_t(w)$  as  $MC_t$ .

The relation between the wholesale goods and the final goods has been shown in Eq.(12). Under the constraint of the demand function, Eq. (15), they maximize their profit, written as

$$\max_{\{P_t(w)\}} \mathbf{E}_t \sum_{j=0}^{\infty} (\xi_p \beta)^j \frac{\Lambda_{t+j}^C}{\Lambda_t^C} \left\{ \frac{P_t(w)}{P_{t+j}} \prod_{k=1}^j \pi_{t+k-1}^{\gamma_p} \pi^{1-\gamma_p} - MC_{t+j} \right\} Y_{t+j}(w), \quad (18)$$

where a fraction  $\xi_p \in [0, 1]$  of the firms follow the index rule,  $P_t^{\gamma_p} = P_{t-1}^{\gamma_p} \pi^{1-\gamma_p}$ , since they do not have the chance to optimize the price. The remaining firms,  $1 - \xi_p$ , can optimize their price setting based on the profit function.  $\Lambda_t^C$  is the marginal utility of households in terms of consumption, since the profit is measured from retail goods, regarded as the numeraire.

By solving the above profit maximization problem, we obtain the FOCs, which are referred to as the price version of the new Keynesian Phillips curve consisting of the following four equations:

$$\frac{\pi_t^o}{\pi_t} = (1 + \lambda_p) \frac{K_t^p}{F_t^p}, \quad (19)$$

$$K_t^p = MC_t Y_t + \xi_p \mathbf{E}_t \Lambda_{t|t+1} \left( \frac{\pi_t^{\gamma_p} \pi^{1-\gamma_p}}{\pi_{t+1}} \right)^{-\frac{1+\lambda_p}{\lambda_p}} K_{t+1}^p, \quad (20)$$

$$F_t^p = Y_t + \xi_p \mathbf{E}_t \Lambda_{t|t+1} \left( \frac{\pi_t^{\gamma_p} \pi^{1-\gamma_p}}{\pi_{t+1}} \right)^{-\frac{1}{\lambda_p}} F_{t+1}^p, \quad (21)$$

$$1 = \xi_p \left( \frac{\pi_{t-1}^{\gamma_p} \pi^{1-\gamma_p}}{\pi_t} \right)^{-\frac{1}{\lambda_p}} + (1 - \xi_p) \left( \frac{\pi_t^o}{\pi_t} \right)^{-\frac{1}{\lambda_p}}, \quad (22)$$

where  $K_t^p$  and  $F_t^p$  denote the auxiliary variables defined recursively as the above Eq.(20) and Eq(21).  $\pi_t^o$  is the target price deviation from the actual price  $\pi_t^o = P_t^o/P_t$ , and the target price  $P_t^o$  is the optimal value derived from the price setting if the wholesalers are given the chance to change their price.

### Final Goods Firms

There is a continuum of final goods firms, indexed by  $f \in [0, V_{t-1}]$ . They produce final goods  $X_{f,t}$  by bundling a set of intermediate goods  $X_{f,t}(h)$  indexed by  $h \in [0, 1]$ . Then the final goods are sold to wholesale goods firms with nominal rigidity due to monopolistic competition. Under the constraint of the CES production function (13), a final goods firm maximizes its profit, given as

$$\max_{\{X_{f,t}(h)\}} P_{f,t}^x X_{f,t} - \int_0^1 P_{f,t}^x(h) X_{f,t}(h) dh, \quad (23)$$

where  $P_{f,t}^x$  and  $P_{f,t}^x(h)$  denote the prices of the final goods and the intermediate goods, respectively. The FOC of the final goods firm indicates the demand function of the intermediate goods, given as

$$X_{f,t} = \left( \frac{P_{f,t}^x(h)}{P_{f,t}^x} \right)^{-\frac{1+\lambda_x}{\lambda_x}} X_{f,t}(h), \quad (24)$$

In addition, we obtain the price equation of the final goods as

$$P_{f,t}^x = \left( \int_0^1 P_{f,t}^x(h)^{-\frac{1}{\lambda_x}} dh \right)^{-\lambda_x}. \quad (25)$$

### Intermediate Goods Firms

There is a continuum of intermediate goods firms, indexed by  $h \in [0, 1]$ , each of which is produced using labor,  $l_t(h)$ , the physical capital stock service,  $u_t^k(h)K_{t-1}(h)$ , and the variety of all final goods,  $V_t$ , where  $u_t^k$  is the capital utilization rate. We assume that the stock of adopted innovation  $V_t$  is also assumed to be the variety of the final goods accumulated from new products.

We assume that each type of firm  $h$  adopts the Cobb-Douglas production function, that is,  $(u_t^k(h) K_{t-1}(h))^\alpha (l_t(h))^{1-\alpha} = \int_0^{V_{t-1}} X_{f,t}(h) df$ , and that a new product,  $\Delta V_t$ , acquired from the adopters increases the quantity of  $X_{f,t}(h)$  on the aggregated level, such as  $(u_t^k K_{t-1})^\alpha (l_t)^{1-\alpha} = \int_0^1 \int_0^{V_{t-1}} X_{f,t}(h) df dh$ , even though it uses the same levels of inputs,  $K_t(h)$  and  $l(h)$ . By substituting Eq.(24) into it, we obtain the aggregated production function including adopted innovation as

$$\left( u_t^k(h) K_{t-1}(h) \right)^\alpha (l_t(h))^{1-\alpha} = \int_0^{V_{t-1}} \left( \frac{P_{f,t}^x(h)}{P_{f,t}^x} \right)^{-\frac{1+\lambda_x}{\lambda_x}} X_{f,t} df. \quad (26)$$

The depreciation rate of the capital also includes the adjustment cost, and it is written as

$$\delta(u_t^k) \equiv \delta_k + b_k \frac{(u_t^k)^{1+\zeta_k}}{1+\zeta_k}, \quad (27)$$

where  $\delta_k$  and  $b_k$  represent the scale parameters of the capital depreciation function and meet the equations  $\delta_k = \delta - \frac{b_k}{1+\zeta_k}$  and  $b_k = \frac{\mu^A}{\beta} + \delta - 1$ , respectively. The dynamic of the capital accumulation is standard:

$$K_t = \left( 1 - \delta(u_t^k) \right) K_{t-1} + z_t^i S(I_t/I_{t-1}) I_t \quad (28)$$

where  $z_t^i$  is the measured productivity of investment, which is referred to as the AR (1) process of the investment efficiency shock, and  $S(\cdot)$  is an adjustment cost function with respect to investment. In addition, the variety of intermediate goods accumulated by the adopted innovation has a dynamic such as

$$V_t = (1 - \delta_a) V_{t-1} + \{1 - S^a(\Delta_{a,t}/\Delta_{a,t-1})\} \Delta_{a,t}, \quad (29)$$

where  $\delta_a$  is the obsolescence rate of the stock and  $S^a(\cdot)$  is a monotonically increasing function of the adjustment cost with respect to the difference of volume of the R&D variety goods. The intermediate firm has to buy new adopted innovation  $\Delta_{a,t}$  to compensate for this obsolescence. Accordingly, by summing up all the activities of the firm, we express its budget constraint as

$$W_t(h) l_t(h) + r_t^k u_t^k(h) K_{t-1}(h) + P_t^V \Delta_{a,t}(h) = \int_0^{V_{t-1}(h)} \frac{P_{f,t}^x(h)}{P_t} X_{f,t}(h) df, \quad (30)$$

where the LHS and the RHS denote the cost and the revenue of the firms, respectively.

The intermediate goods firms maximize the net present value of the profit by controlling the price of intermediate goods,  $P_{f,t}^x(h)$ , the capital stock,  $K_t(h)$ , the capital utilization rate,  $u_t^k(h)$ , the labor demand,  $l_t(h)$ , the new product stock,  $V_t(h)$ , and its current dividend  $D_t$ , so their optimization problem is obtained as

$$\max_{\{P_{h,t}^x, K_t, u_t^k, l_t, V_t, D_t\}} \mathbf{E}_t \sum_{j=0}^{\infty} \beta^j \frac{\Lambda_{t+j}^C}{\Lambda_t^C} \left[ \int_0^{V_{t-1}(h)} \frac{P_{f,t}^x(h)}{P_t} X_{f,t}(h) df - W_t(h) l_t(h) \right. \\ \left. - r_t^k u_t^k(h) K_{t-1}(h) - P_t^V \Delta_{a,t}(h) \right], \quad (31)$$

subject to Eq.(26) through Eq.(30). The FOCs of the optimization problem consist of five equations, as shown below.

Firstly, from the viewpoint of optimal pricing, the intermediate goods price meets

$$\frac{P_{f,t}^x(h)}{P_t} = (1 + \lambda_x) s_t. \quad (32)$$

where  $s_t$  is the Lagrangian multiplier in the production function Eq.(26), and it means the shadow price of intermediate goods or the relative price of intermediate goods to retail goods, since the retail goods price is the numeraire in this paper.

Secondly, the FOC with respect to fixed capital rental cost,  $r_t^k$ , is given as

$$r_t^k = \alpha s_t \left( \frac{u_t^k K_{t-1}}{l_t} \right)^{\alpha-1}, \quad (33)$$

where the effective capital equipment ratio,  $\frac{u_t^k K_{t-1}}{l_t}$ , is common to all intermediate firms, so we omit the index  $h$  for simplicity.

Thirdly, the FOC of the labor demand,  $l_t(h)$ , is obtained as

$$W_t = (1 - \alpha) s_t \left( \frac{u_t^k K_{t-1}}{l_t} \right)^\alpha. \quad (34)$$

Fourthly, the FOC with respect to newly adopted innovation,  $\Delta_{a,t}$ , is written as

$$P_t^V = \Gamma_t^a \left\{ 1 - S'_a \left( \frac{\Delta_{a,t}(h)}{\mu^V \Delta_{a,t-1}(h)} \right) - S'_a \left( \frac{\Delta_{a,t}(h)}{\mu^V \Delta_{a,t-1}(h)} \right) \frac{\Delta_{a,t}(h)}{\mu^V \Delta_{a,t-1}(h)} \right\} \\ + \mu^V \mathbf{E}_t A_{t|t+1} \Gamma_{t+1}^a S'_a \left( \frac{\Delta_{a,t+1}(h)}{\mu^V \Delta_{a,t}(h)} \right) \left( \frac{\Delta_{a,t+1}(h)}{\mu^V \Delta_{a,t}(h)} \right)^2, \quad (35)$$

where  $\Gamma_t^a$  is the Lagrangian multiplier of the firm's accumulation of newly adopted innovation. The FOC with respect to the stock of goods variety is obtained as

$$\Gamma_t^a = \mathbf{E}_t A_{t|t+1} \left[ (1 - \delta_a) \Gamma_{t+1}^a + \lambda_x s_{t+1} \frac{(u_{t+1}^k K_t)^\alpha (l_{t+1})^{1-\alpha}}{V_t} \right], \quad (36)$$

where  $P_t^V$  is the value of the adopted innovation which is also used in the adopter's value function (5) described in the previous subsection.

We omit the price dispersion of wholesale goods, unlike Ikeda and Kurozumi (2014), since we have to use a log-linearized DSGE model to estimate it rather than a non-linearized one. Hence, the quantity of aggregated final goods is equal to those of the wholesale goods and retail goods, such as  $X_{f,t} = Y_t(w) = Y_t$ . Finally, after substituting  $X_{f,t}(w) = Y_t$  and Eq.(7) into the Cobb-Douglas production function,  $(u_t^k K_{t-1})^\alpha (A_{t-1} l_t)^{1-\alpha} =$

$X_{f,t}$ , we obtain

$$Y_t = \left( u_t^k K_{t-1} \right)_t^\alpha \left( V_{t-1}^{\frac{\lambda_x}{1-\alpha}} l_t \right)^{1-\alpha}, \quad (37)$$

The appendix shows the derivation of the production function (37).

### 3.3 Households

There is a continuum of households, indexed by  $g \in [0, 1]$ . However, when the households face problems maximizing their intertemporal utility, they are regarded as a representative agent that attains utility from consumption and leisure. Households' preference is given as

$$\max_{\{C_t, B_t, K_t, I_t\}} \mathbf{E}_0 \beta^t z_t^b \sum_{t=0}^{\infty} \left\{ \ln(C_t - h C_{t-1}) - z_t^l \gamma l \frac{l_t^{1+\omega_l}}{1+\omega_l} \right\}, \quad (38)$$

where  $C_t$  and  $l_t$  denote the aggregate consumption and labor supply, respectively. We also allow for habit persistence in their preference by adding  $h C_{t-1}$ .  $z_t^b$  and  $z_t^l$  are the preference shock and labor supply shock, respectively. The budget constraint of households is given as

$$C_t + I_t + \frac{B_t}{P_t} + \Pi_t = r_t^k u_t^k K_{t-1} + W_t l_t + r_{t-1}^n \frac{B_{t-1}}{P_t} + T_t, \quad (39)$$

where  $B_t$ ,  $T_t$  and  $\Pi_t$  are the bond holding and the lump sum public transfer and dividend (or profit) of the productive sector, respectively.  $W_t$  is the real wage. Accordingly, by solving the above problem, the first-order condition (FOC) in terms of consumption is given as

$$\Lambda_t^C = \frac{z_t^b}{C_t - h C_{t-1}} - \beta h \mathbf{E}_t \frac{z_{t+1}^b}{C_{t+1} - h C_t}, \quad (40)$$

where  $\Lambda_t^C$  is the marginal utility with respect to consumption. In a similar way, the FOCs in terms of bonds are obtained as

$$\Lambda_{t|t+1} \equiv \beta \frac{\mathbf{E}_t \Lambda_{t+1}^C}{\Lambda_t^C}, \quad (41)$$

$$\Lambda_{t|t+1} = \frac{\mathbf{E}_t \pi_{t+1}}{r_t^n}, \quad (42)$$

where  $\Lambda_{t|t+1}$  is the marginal utility with respect to bonds. Eq. (41) shows that  $\Lambda_{t|t+1}$  is the bond price measured by the shadow price of consumption goods, and it is also defined as a stochastic discount factor (SDF). Eq.(42) is regarded as the Euler equation for consumption.

Households are also fixed capital holders, and they optimize the capital holding,  $K_t$ , investment,  $I_t$ , and capital utilization rate,  $u_t^k$ , to maximize the intertemporal utility. The FOCs in terms of fixed capital and investment are given as

$$q_t^k = \mathbf{E}_t \Lambda_{t|t+1} \left[ r_{t+1}^k u_{t+1}^k + q_{t+1}^k \left\{ 1 - \delta \left( u_{t+1}^k \right) \right\} \right] \quad (43)$$

$$1 = z_t^i q_t^k \left\{ 1 - S \left( \frac{I_t}{I_{t-1}} \right) - S' \left( \frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right\} + \mathbf{E}_t \Lambda_{t|t+1} z_{t+1}^i q_{t+1}^k S' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \quad (44)$$

$$r_t^k = q_t^k \delta' \left( u_t^k \right) \quad (45)$$

where  $q_t^k$  is the Tobin's Q, which satisfies  $q_t^k \equiv \frac{\Lambda_t^k}{\Lambda_t^C}$ .

### Wage Setting

The households indexed by  $g \in [0, 1]$  face monopolistic competition for supplying their specialized labor, and their wage-setting problem possesses the property of nominal rigidity, following Erceg, Henderson and Levin (2000). When households decide their wage with the intermediate goods firms, they maximize the present value of the stream of their utilities as

$$\max_{\{W_{g,t}\}} \mathbf{E}_t \sum_{j=0}^{\infty} (\beta \xi_w)^j \left\{ \Lambda_{t+j}^C l_{g,t+j} (\mu^A)^j W_{g,t} \prod_{k=1}^j \left( \frac{\pi_{t+k-1}^{\gamma_w} \pi^{1-\gamma_w}}{\pi_{t+k}} \right) - \gamma_l z_{t+j}^b z_{t+j}^l \frac{l_{g,t+j}^{1+\omega_l}}{1+\omega_l} \right\}, \quad (46)$$

where a fraction of  $\xi_w \in [0, 1]$  of households follow an index rule,  $P_t^{\gamma_w} W_{g,t} = P_{t-1}^{\gamma_w} W_{g,t-1} \pi^{1-\gamma_w}$ , since they do not have the chance to optimize their wage. The remaining households,  $1 - \xi_w$ , conduct optimal wage setting.  $\mu^A$  is the steady state of the technology progress rate. The link between the specialized labor supply of each type of household and the aggregate labor supply is given by

$$l_{g,t} = \left( \frac{W_{g,t}}{W_t} \right)^{-\frac{1+\lambda_w}{\lambda_w}} l_t, \quad (47)$$

where  $\lambda_w > 0$  is the net markup rate of the real wage. By solving the above maximization problem, we obtain the FOCs, which are referred to as the wage version of the new Keynesian Phillips curve consisting of the following four equations:

$$\frac{W_t^o}{W_t} = \left\{ (1 + \lambda_w z_t^w) \frac{K_t^w}{F_t^w} \right\}^{\frac{\lambda_w}{\lambda_w + \omega + \lambda_w \omega}}, \quad (48)$$

$$K_t^w = \gamma_l z_t^b z_t^l l_t^{1+\omega} + \mathbf{E}_t \beta \xi_w \left( \mu^A \frac{\pi_{t-1}^{\gamma_w} \pi^{1-\gamma_w}}{\pi_t} \frac{W_t}{W_{t+1}} \right)^{-\frac{1+\lambda_w}{\lambda_w} (1+\omega)} K_{t+1}^w, \quad (49)$$

$$F_t^w = \Lambda_t^C W_t l_t + \mathbf{E}_t \beta \xi_w \left( \mu^A \frac{\pi_{t-1}^{\gamma_w} \pi^{1-\gamma_w}}{\pi_t} \frac{W_t}{W_{t+1}} \right)^{-\frac{1}{\lambda_w}} F_{t+1}^w, \quad (50)$$

$$1 = \xi_w \left( \mu^A \frac{\pi_{t-1}^{\gamma_w} \pi^{1-\gamma_w}}{\pi_t} \frac{W_t}{W_{t+1}} \right)^{-\frac{1}{\lambda_w}} + (1 - \xi_w) \left( \frac{W_t^o}{W_t} \right)^{-\frac{1}{\lambda_w}}, \quad (51)$$

where  $W_t^o$  is the target wage, which is the optimal solution derived from the wage-setting problem when the households are given the chance to change their wage, and we add a wage markup shock,  $z_t^w$ , to the RHS of Eq.(48).

### 3.4 Other Equations

#### Market Clearing Condition

The aggregate output in the whole economy is composed of the sum of the demand for the retail goods. The market-clearing condition of the retail goods is given as

$$Y_t = C_t + I_t + I_{a,t} (Z_{t-1} - V_{t-1}) + I_{a,t} + \overline{g/y} z_t^g, \quad (52)$$

where the third term is the investment of the adopters and the fourth term is the R&D investment of the innovators.  $z_t^g$  denotes exogenous expenditure, such as the government sector. However, each term in the equation does not necessarily match the data that we use for estimation.

Since the real GDP with the benchmark year 2005 does not contain the R&D investment as its component, we use the following definition of output in the observation equation explained in Section 4:

$$\tilde{Y}_t = C_t + I_t + \overline{g/y} z_t^g. \quad (53)$$

#### Monetary Policy

The central bank follows a Taylor-type monetary policy rule given as

$$\ln r_t^n = \phi_r \ln r_{t-1}^n + (1 - \phi_r) \left\{ \ln r_n + \phi_\pi \ln \left( \frac{\pi_t}{\pi} \right) + \phi_y \ln \left( \frac{y_t}{y} \right) \right\} + \varepsilon_t^r, \quad (54)$$

where  $\phi_r \in [0, 1)$  denotes the degree of policy rate smoothing.  $\phi_\pi$  and  $\phi_y$  are policy responses to inflation and output, respectively.  $\varepsilon_t^r$  is a monetary policy shock with an iid process.



## Equilibrium Conditions and Structural Shocks

To acquire an equilibrium of the model and to estimate it, we use equations such as Eq.(1) through Eq.(3), Eq.(6) through Eq.(10), Eq.(19) through Eq.(22), Eq.(26) through Eq.(30), Eq.(32) through Eq.(37), Eq.(40) through Eq.(42), and Eq.(48) through Eq.(54). There are eight structural shocks, of which all shocks except the monetary policy shock follow the AR (1) process. The appendix presents the conditions and the shocks.

### 3.5 Two Alternative Models

To investigate the properties of the decomposition obtained by our model, we introduce two alternative models. One is a “model without nominal rigidities” from which we remove the assumptions of nominal rigidities in price and wage setting from the above original model by setting the Calvo parameters of both nominal rigidities to zero. The other model is a standard New Keynesian model, which replaces the exogenous growth rate,  $\mu_t^z$ , by removing the assumption of endogenous growth explained in Section 3.1 from the original model. The law of motion of the exogenous growth rate is defined as

$$\log(\mu_t^z/\mu^z) = \rho_z \log(\mu_{t-1}^z/\mu^z) + \varepsilon_t^z \quad (55)$$

where  $\mu^z$  is the steady-state growth rate corresponding to  $\mu^A$  in the endogenous growth model and  $\varepsilon_t^z$  is a technology shock following an iid process. Hereafter, we refer to this model as the “standard NK model.”

Thus, we compare the properties of the technology progress rate in the endogenous growth New Keynesian model (or “benchmark mode” in our paper) with that in the standard NK model to evaluate the performance of the endogenous growth mechanism as the low-pass filter extracting the common trend component of economic fluctuations.

## 4 Estimation Strategy

This section describes the estimation strategy, including the data and the link between the endogenous and the observable variables.

### 4.1 Estimation Methods

In this paper the values of the model parameters are estimated following a Bayesian approach via Markov chain Monte Carlo (MCMC) simulation. We use a stylized solution method for estimation; specifically, we log-linearize the model shown above and convert it into a linear Gaussian state-space model after detrending the endogenous variables around their steady states. Then we evaluate the posterior density by combining the

value of the likelihood obtained from the Kalman filter with a prior density. We generate 3 chains composed of 125,000 draws from the posterior distribution of the parameters by the Metropolis-Hastings algorithm and discard the first 20 percent of them (i.e. 20,000 draws) as burn-in iterations.

## 4.2 Observable Variables and Data

As can be seen from Table 1, we adopt seven observed variables – (1) output growth, (2) consumption, (3) investment, (4) retail goods price, (5) real wage, (6) labor supply and (7) nominal interest rate, for Japan from 1980:Q2 through 2013:Q4. We collect the real GDP,  $Y_t^{data}$ , real private consumption,  $C_t^{data}$ , and fixed capital formation,  $I_t^{data}$ , from the Cabinet Office’s National Accounts as output, consumption and investment. We use the 2005 benchmark data that cover the period 1980:Q1–2013:Q4.

To make these demand series per capita, we divide them by the labor force,  $N_t^{obs}$ . As the price level, we use the implicit GDP deflator index from the Cabinet Office. The worked hour indices,  $l_t^{obs}$ , and the real wage indices,  $W_t^{obs}$ , of the Monthly Labor Survey are used for the labor supply,  $l_t$ , and real wage,  $W_t$ . As the nominal interest rate, we use the Bank of Japan’s secured overnight call rate.

[ Insert Table 1 ]

## 4.3 Link between Observable and Endogenous Variables

The equilibrium conditions of the model are rewritten in terms of detrended variables around the steady state, and the detrended variables are given as  $y_t = \frac{Y_t}{A_{t-1}}$ ,  $\tilde{Y}_t = \frac{\tilde{Y}_t}{A_{t-1}}$ ,  $c_t = \frac{C_t}{A_{t-1}}$ ,  $i_t = \frac{I_t}{A_{t-1}}$ ,  $w_t^o = \frac{W_t^o}{A_{t-1}}$ ,  $w_t = \frac{W_t}{A_{t-1}}$ ,  $k_t = \frac{K_t}{A_{t-1}}$ ,  $mc_t = \frac{MC_t}{(A_{t-1})^{1-\alpha}}$ ,  $s_t = \frac{S_t}{(A_{t-1})^{1-\alpha}}$ ,  $p_t^V = \frac{P_t^V V_{t-1}}{A_{t-1}}$ ,  $i_{d,t} = \frac{I_{d,t}}{A_{t-1}}$ ,  $\lambda_t^c = \Lambda_t^C A_{t-1}$ ,  $\gamma_{a,t} = \Gamma_{a,t} \frac{V_{t-1}}{A_{t-1}}$ ,  $\tilde{\Delta}_{a,t} = \frac{\Delta_{a,t}}{V_{t-1}}$ ,  $a_{t-1} = \frac{V_{t-1}}{Z_{t-1}}$ ,  $i_{a,t} = \frac{I_{a,t} V_{t-1}}{A_{t-1}}$ ,  $j_t = \frac{J_t V_{t-1}}{A_{t-1}}$ ,  $\phi_t = \frac{\Phi_t A_{t-1}}{V_{t-1}}$ ,  $k_t^p = \frac{K_t}{A_{t-1}}$ ,  $f_t^p = \frac{F_t}{A_{t-1}}$ ,  $k_t^w = \frac{K_t}{A_{t-1}}$ ,  $f_t^w = \frac{F_t}{A_{t-1}}$ ,  $\mu_t^A = \frac{A_t}{A_{t-1}}$ , and  $\mu_t^V = \frac{V_t}{V_{t-1}}$ .

The links between the observable and the endogenous variables are given as follows:

### 1. Real GDP Growth Rate

$$\Delta Y_t^{obs} = \frac{Y_t^{data}/N_t^{data}}{Y_{t-1}^{data}/N_{t-1}^{data}} = \mu_{t-1}^A \times \frac{\tilde{Y}_t}{\tilde{Y}_{t-1}}. \quad (56)$$

### 2. Real Private Consumption Growth Rate

$$\Delta C_t^{obs} = \frac{C_t^{data}/N_t^{data}}{C_{t-1}^{data}/N_{t-1}^{data}} = \mu_{t-1}^A \times \frac{c_t}{c_{t-1}}. \quad (57)$$

### 3. Real Private Investment Growth Rate

$$\Delta I_t^{obs} = \frac{I_t^{data}/N_t^{data}}{I_{t-1}^{data}/N_{t-1}^{data}} = \mu_{t-1}^A \times \frac{i_t}{i_{t-1}}. \quad (58)$$

### 4. Real Wage Growth Rate

$$\Delta W_t^{obs} = \frac{W_t^{data}}{W_{t-1}^{data}} = \mu_{t-1}^A \times \frac{w_t}{w_{t-1}}. \quad (59)$$

### 5. Labor Supply

$$l_t^{obs} = \frac{l_t^{data}}{100} = \bar{l} \times l_t. \quad (60)$$

### 6. Nominal Interest Rate

$$r_t^{n,obs} = 1 + \frac{R_t^{n,data}}{400} = r_t^n. \quad (61)$$

### 7. Price Inflation Rate

$$\pi_t^{obs} \equiv \frac{P_t^{data}}{P_{t-1}^{data}} = \pi_t. \quad (62)$$

The first four observable variables are the first difference of the data.  $\mu_t^A$  is the logarithm of the common growth rate, that is,  $\log(A_t/A_{t-1})$ . The annualized nominal rate is changed to a quarterly basis by dividing it by 400. Notice that the first four equations indicate that those variables have a long, stable relation with the stochastic common trend,  $A_t$ , as described in Section 2.2.

## 4.4 Calibrated Parameters and Prior Distribution

In this model we fix 11 parameters in Table 2 to avoid identification problems. The steady state of the exogenous demand share of the output, including government,  $\overline{g/y}$ , is set to 0.25. The depreciation rates of the physical capital stock,  $\delta$ , and the capital share,  $\alpha$ , are set to 0.025 and 0.4, respectively. The subjective discount rate,  $\beta$ , is closer to 1 than in recent studies of the Japanese economy, since the monetary policy rate has been permanently rather than temporarily close to 0 since 1999:Q1.

The prior distribution of the parameters to be estimated is described in the third through the fifth column of Table 3.

[ Insert Table 2 ]

[ Insert Table 3 ]

## 5 Empirical Results and Discussion

### 5.1 Estimated Parameters

The estimations of the parameters of the benchmark model as well as the two alternative models are summarized in Table 3. To focus on growth as well as business cycles, we expand the sample period to over thirty years, although it includes the period of the zero interest rate policy (ZIRP). According to Hirose and Inoue (2016), the estimation bias of the ZIRP is not large and is acceptable when adopting a Taylor-type linear monetary policy rule. To test the convergence of the MCMC sampling, we conduct Gelman and Rubin (1992)'s convergence diagnostic and confirm the convergence of all the parameters, and we compare the empirical results of our benchmark model with those of the standard NK model, the growth part of which is mainly based on that of Altig et al. (2011).

There are three remarks to make. First, we observe very similar values in the common parameters of the three models except for the following results. The coefficient of the investment adjustment cost is around 3.4 in the benchmark model, while it is 6.0 in the no nominal rigidity model and 8.3 in the NK model. This is because the common trend,  $\mu_{t-1}^A$ , of the benchmark model moves more smoothly than that of the NK model. Larger adjustment costs are more likely to work to offset fluctuations in the common trend growth. In the no nominal rigidity model, it is observed that the inverse of the elasticity of the labor supply is 37.2 and the parameter of monetary policy smoothing is 0.1, since the classical dichotomy is held in the no nominal rigidity model, in which both the Calvo price and the Calvo wage parameter are set to zero. The assumption causes the monetary policy to be ineffective, as indicated by the variance decomposition of the business cycles in Table 6.

Second, the standard deviation of the success probability shock in the two endogenous growth models is similar to that of the TFP shock in the NK model, although the persistence of the former shock is a high value, such as 0.92 to 0.97, whereas that of the latter shock is 0.28. These differences affect the difference in the variance decomposition of the business cycles between the three models, as described later in Table 6.

Third, we compare the estimation of our three models with previous Japanese studies, such as those by Sugo and Ueda (2008) and Kaihatsu and Kurozumi (2014), who limit the sample period to before the ZIRP. The Calvo price parameter of this study is nearly 0.96, and the Calvo wage parameter is around 0.65, whereas Sugo and Ueda (2008) estimate them to be 0.88 and 0.52 and Kaihatsu and Kurozumi (2014) estimate them to be 0.68 and 0.50, respectively. Through 3 studies focusing on Japan, we see that the Calvo price parameter is higher than the Calvo wage parameter, but the scale itself is likely to be inconsistent. Habit formation of consumption is another controversial estimate. Our

result is 0.96, while in Sugo and Ueda (2008) it is 0.10 and in Kaihatsu and Kurozumi (2014) it is 0.26. This discrepancy might come from differences in the procedure followed for detrending the data and the sample period. Sugo and Ueda (2008) uses the HP filter for detrending, whereas we do not use such a filter. The fluctuations in inflation are observed to be much smaller during the period of the ZIRP than during other periods, which make habit formation more persistent under a constant price level.

Table 4 shows the marginal likelihoods of the three models. When we use even prior model odds for the three models, the posterior odds of the benchmark model to the no nominal rigidity model are one, which suggests that the nominal rigidities of price and wage are significant. However, when we expand the model selection to a choice between three models, the NK model overwhelmingly dominates the other models with an endogenous growth framework. The smoothness of each trend estimated by the models with endogenous growth, which can be seen in panel (a) of Figure 2, might worsen the data fitting.

[ Insert Table 3 and Table 4 ]

## 5.2 Impulse Response

Before comparing the business cycle and common trend components of each model, we check the impulse response of the endogenous variables to a success rate shock. Figure 1 shows the impulse responses of endogenous variables to a one standard deviation success rate shock: the solid lines are those of the benchmark model and the dash-dotted lines are those of the model without nominal rigidity. All the parameters are set to the posterior mean described in Table 3.

[ Insert Fig. 1 ]

Panel (a) shows the impulse responses of the detrended variables,  $\hat{y}_t$ ,  $\hat{c}_t$ ,  $\hat{i}_t$ ,  $\hat{l}_t$ ,  $\hat{i}_t^{RD}$  and  $\hat{\mu}_t$ , where R&D investment,  $i_t^{RD}$ , is defined as

$$i_t^{RD} = i_t^a \left( \frac{1}{a_{t-1}} - 1 \right) + i_t^d \quad (63)$$

A success rate shock increases the goods variety,  $V_t$ , as can be seen from Eq.(8), and hence the level of technology rises. This effect is reflected in the increase in the technology progress rate,  $\mu_t$ . Besides this direct effect, a success rate shock stimulates R&D investment, as an increase in adoption due to the improvement of the success probability,  $\lambda_t$ , decreases the stock of the idea  $Z_t$  and increases the value of innovation. The response of labor differs greatly between the two models. The reason why labor does

not react with the model without nominal rigidity depends on the estimated parameters rather than the characteristics of the model. The labor supply in the benchmark model changes to the positive direction in the short run in response to the increase in productivity, as in the conventional wage rigidity model.

In Panel (a) it seems that  $\hat{y}_t$ ,  $\hat{c}_t$  and  $\hat{i}_t$  move irrelevantly and there is no co-movement at first glance, but notice that these are detrended variables. Panel (b) shows the deviation of  $\hat{Y}_t$ ,  $\hat{C}_t$ ,  $\hat{I}_t$  and  $\hat{I}_t^{RD}$  from the balanced growth path (trend of growth) when a success rate shock has occurred. The cumulative amount of  $\mu_t^A$  is added to each series of Panel (a). From the perspective of the deviation from the balanced growth path, the increase in the rate of technological progress caused by a success rate shock increases the output, as is usually considered. It is also confirmed that consumption and investment co-move with the output, suggesting that the success rate shock of our model plays a role similar to the TFP shock of a conventional model. However, in addition to the direct effect of increasing the goods variety, there is an indirect effect of stimulating R&D investment.

### 5.3 Business Cycle and Common Trend Components

We evaluate the business cycle components and trend components of output, consumption and investment derived from our estimation of the three DSGE models by comparing them with those of the Hodrick-Prescott (HP) filter and band-pass (BP) filter, which detrend the univariate time series<sup>2</sup>. Table 5 summarizes the standard deviations and correlations of the business cycle components of these five approaches. Figures 2 and 3 also depict the decomposition of both trend and cycle components. The business cycle components of output, consumption and investment in the models are defined as  $\hat{y}_t$ ,  $\hat{c}_t$  and  $\hat{i}_t$ , and the trend component is defined as  $\mu_t^A$ .

As can be seen from panel (a) of Figure 2, the cycle components of the output in the three DSGE models are highly aligned with one another, especially between the benchmark and the no nominal rigidity model. The second column of Table 5 (b) also shows these high correlations. In addition, the peaks and troughs of these components seem to coincide with the recessions reported by the ESRI marked as green shaded areas. It indicates that the business cycle components of the three DSGE models are successfully extracted from the data. On the other hand, panel (b) depicts the common trend components of the real GDP in terms of the five approaches. While the trends of both the HP and the BP filter pass through the center of the fluctuations of the actual GDP, those of the DSGE model swing much higher and lower than the actual values in some periods. In particular, from 1984 to 1999, the trends locate upwards away from the real GDP,

<sup>2</sup>We adopt the BP filter proposed by Christiano and Fitzgerald (2003) referred to as asymmetric filtering with a one-side moving average, with which we can obtain the latest values of the cycle component

whereas they locate downwards away from it during 2002 to 2007. These movements are derived from big movements of the trend of investment, since the DSGE models impose the balanced growth theory between output and investment. According to the DSGE models, the decline in R&D activities or TFP from 2002 to 2007 induces a huge drop in the common trend. As a result, exponential expansions of the business cycles are observed in this period, as shown in panel (a).

The four panels of Figure 3 show the business cycle components of the real GDP, consumption and investment extracted from the benchmark model and the HP and BP filters as well as the coincident business cycle index (CI) reported by the ESRI, respectively. The blue solid lines marked with diamond symbols represent the cycles of the benchmark, while the red solid lines with asterisk symbols and the black dashed lines are those extracted from the HP and BP filters, respectively. As can be seen from panels (b) through (d), the peaks and troughs of the cycles of the three variables seem to match the recessions represented by the green shaded areas. As in panel (a), the fluctuations of the three variables of the benchmark model are likely to be highly consistent with those of the CI. Although the cycles of investment of the benchmark model are very similar to those of both the filters, the cycles of output and consumption extracted from the DSGE model fluctuate much more than those of the filters. Again, these characteristic movements are generated from the balanced growth theory working among the three macroeconomic variables. In contrast, both the filters independently extract the cycle components by using only univariate information from each variable. These features, such as the correlation among the five approaches, are also summarized in panels (b) through (d) in Table 5.

[ Insert Fig. 2 and Fig. 3 ]

[ Insert Table 5 ]

## 5.4 Variance and Historical Decompositions

### 5.4.1 Variance Decomposition Analysis

Justiniano, Primiceri and Tambalotti (2010) conducts unconditional variance decomposition regarding standard business cycle frequencies from 6 to 32 quarters. We follow this approach and apply it to longer cycles including trend components regarding frequency over 32 quarters in Table 6 in addition to the standard business cycles, as shown in Table 7. This analysis sheds light on the role of shocks in medium-term cycles as well as short-term cycles.

There are two remarks to make. First, by comparing the variance decomposition for longer cycles (Table 6) with that for business cycles (Table 7), the contributions of

the R&D success probability shock of the two DSGE models with endogenous growth account for a much larger portion of the variances of the common trend than in the case of the cycle components for all three variables. For example, in the benchmark model, the success probability shock contributes 29.4 % of the trend of output but only 3.6 % of the cycles of output. Surprisingly, in the case of the no nominal rigidity model, this shock explains about 86.5 % of the trend of the output although only 14.0 % of the cycles. This contrast between the two different frequencies might depend on the impact of the change in the R&D activities' efficiency generated by the success probability shock. This change influences the growth rate strongly but scarcely affects the business cycles. Similarly, in the case of the standard NK model, the TFP shock contributes a larger ratio of the variances of the common trend than that of the cycles of all three variables. However, the TFP shock accounts for large amounts of around 19 % to 68 % for all three variables even in the short-term cycles. Accordingly, the success probability shock has different features from the TFP shock in both frequencies.

Second, the contribution of the monetary policy shock in the benchmark model is very large in both frequencies regarding all three variables, as shown in Table 6 (a) and Table 7 (a). Similarly, the monetary policy shock accounts for a high percentage of both trend and cycle components even in the standard NK model. These results stem largely from the strong nominal rigidities suggested by the high Calvo price parameter, such as nearly 0.96, and the low price index parameter, such as 0.09. In other words, the real variables are directly influenced by the monetary policy shock, since the price level is hardly flexible. However, it is likely that the effect of the monetary policy is overestimated or includes a sort of upper bias, since the latter part of our sample period includes the zero interest rate period. On the other hand, as shown in Table 7 (b), the exogenous demand shock is a main contributor to the cycle components of the three variables in the no nominal rigidity model, since the Calvo price parameter is set to zero. The monetary policy does not contribute to the variance of the real variables, as shown in panel (b) of Tables 6 and 7. Accordingly, the fluctuations in the real variables are mainly explained by the real exogenous demand shock, instead of the monetary policy shock, as a result of assuming the neutrality of money.

[ Insert Table 6 and Table 7 ]

#### 5.4.2 Historical Decomposition Analysis

Before considering the historical decomposition, we mention the property of eight structural shocks. Since  $\log \mu_t^A = \frac{\lambda_x}{1-\alpha} \log \mu_t^V$ , the growth rate of the goods variety,  $\mu_t^V$ , is the



key factor of economic growth,  $\mu_t^A$ , which can be rewritten as

$$\log \mu_t^A = \frac{\lambda_x}{1-\alpha} \log [(1 - \delta_a) + \Delta_{a,t}(z_t^\lambda, z_t^b, z_t^l, z_t^g, z_t^P, z_t^W, z_t^i, \varepsilon_t^R)],$$

where  $\mu_t^V$  consists of the determined growth rate,  $(1 - \delta_a)$ , and the stochastic growth rate,  $\Delta_{a,t}$ , which is generated from the combination of an additional goods variety.  $\Delta_{a,t}$  is a sort of function that is affected directly by the R&D success probability shock  $z_t^\lambda$  and indirectly by the seven other shocks through the market mechanism. Hence, the goods variety shock and the success rate shock have a permanent effect on the four series – output, consumption, investment and wage – via the R&D sector, whereas the remaining shocks must also influence the common growth rate,  $\mu_t^A$ , through the change in  $\Delta_{a,t}$ , although they have a tiny effect on the common growth rate, as shown in the following figures.

### Technology Progress Rate or Common Trend

Figure 4 shows the estimation and historical decomposition of the time series of the common growth rate,  $\mu_t^A$ . Although the ranges of the rate after 1982:Q1 are located between  $-1.5\%$  and  $1.5\%$  in a quarterly period, most of the growth rates are shown to have negative values between 1991:Q1 and 2013:Q1. These values might support that period being referred to as the “lost decades.” Since we find two deep declines in the progress rate during the Asian financial crisis in 1998 and the failure of Lehman Brothers in 2008:Q3, these two crises caused not only big recessions in terms of the business cycle but also stagnation of the economic growth in terms of the supply side.

As shown in Figure 4, the historical decomposition of the common growth rate,  $\mu_t^A$ , shows that the persistence of the positive R&D success rate shock (the area shaded in blue) gradually increased the progress rate for the Plaza Accord<sup>3</sup> in 1985 before the bubble boom started. However, the persistence of the negative R&D success rates dropped the growth rate by reducing a variety of new products after 1990:Q1. This shock also decreased the common growth rate,  $\mu_t^A$ , after 1992:Q1. This figure suggests that a rise (drop) in the R&D success rate induces expansion (shrinkage) of the variety of intermediate goods. The activities of the R&D sector might positively affect the quantity of the output in the productive sector via the fluctuation of converting innovation into intermediate goods variety. Furthermore, sluggish spending of the R&D investment shrunk the variety of new products in the productive sector, and then both the R&D investment and

<sup>3</sup>The Plaza Accord was an agreement between the governments of France, West Germany, Japan, the United States and the United Kingdom to depreciate the U.S. dollar in relation to the Japanese yen and German Deutsche Mark by intervening in the currency markets. The five governments signed the accord on September 22, 1985 at the Plaza Hotel in New York City.

the variety of new products fell by feeding on each other.

After 1997 the stagnation can mainly be explained by the persistent declines in the R&D success rate shock. This might also be regarded as reductions in R&D investment attributed to the financial problems of the corporate sector, although our model does not include the financial sector.

On the other hand, the labor disutility, monetary policy and price markup shocks have very tiny but positive effects on the growth rate, whereas the exogenous demand shock, including the government spending policy, does not affect the growth in the sample period overall.

[ Insert Fig. 4 ]

### **Cycle Components of GDP, Consumption and Investment**

Next we consider the decomposition of the business cycle components of output, consumption and investment, defined as  $\hat{y}_t$ ,  $\hat{c}_t$  and  $\hat{i}_t$ , as shown in panels (a) through (c) of Figure 5, respectively. The investment efficiency, labor disutility and exogenous spending shocks affect the three cycle components in the same direction, while the preference and R&D success rates must have a contrasting effect on investment and consumption with the opposite direction via the substitution effect between consumption and saving.

We consider two shocks, namely the R&D success rate and preference shocks, which have a substitution effect. The former positive shock increased the cycle of investment as well as the common trend until 1991, that is, the end of the "bubble boom," and then the negative shock decreased the cycle after the end of the boom, as shown in Figure 5 (c). This shock, however, had opposite effects on the cycles of consumption, as shown in Figure 5 (b). On the other hand, the latter negative shock sustained a certain level and decreased the consumption after 1991, as presented in Figure 5. This negative preference shock increased investment and induced upward pressure on the output for that period, as shown in Panel (c).

To sum up, the fluctuation in the R&D success rate exerted a strong impact on the long stagnation in the three cycles after the "bubble boom", as shown in the three panels. Again, if we regard the R&D success rate as R&D investment, a reduction in R&D investment also caused a drop of a large fraction of physical investment. However, this reduction contributed to an increase in consumption due to the lower level of the nominal interest rate.

[ Insert Fig. 5 ]

## **Inflation**

Panel (d) in Figure 5 depicts the decomposition of inflation. Although the contribution to inflation is mainly accounted for by the price markup shock and the labor supply shock, the former pulls it downwards and the latter pushes it upwards. Since 1994 a reduction in the success rate of the R&D sector has decreased inflation. The effect of the monetary policy shock is not observed at all.

## **Labor Supply**

Panel (e) in Figure 5 shows the decomposition of the labor supply. As shown in the figure, the labor disutility and preference shocks in addition to the R&D success rate shock are the main sources of variations in the labor supply. Although the successive decline of labor hours after 1990 is explained by the series of negative labor shocks in our model, we are strongly convinced that a drastic change of social institutions and environments actually happened and attacked households, and this must be a factor in the deep decline of the labor supply.

## **6 Conclusion**

In this paper, following Comin and Gertler (2006), we incorporate the endogenous productivity growth framework of Romer (1990) into a medium-scale new Keynesian DSGE model with nominal price and wage rigidities to evaluate the Japanese economy after 1980 for over three decades, including the bubble burst in 1991, the Asian currency crisis in 1998 and the Lehman Brothers' failure in 2008. To measure the performance of our DSGE model, we also build two alternative models, specifically one that excludes the assumption of nominal rigidities and another that is a standard DSGE model with an exogenous TFP shock. Using Bayesian estimation, we decompose the original time series into business cycles and the trend and compare them with those extracted by the HP and BP filters. We find the factors that contributed to the huge declines in output during the three economic crises by calculating the historical decomposition of the trend.

The contributions of this study are twofold. First, we estimate a DSGE model with the framework of R&D endogenous growth proposed by Comin and Gertler (2006) for Japan. In addition, we evaluate this model by implementing the model selection out of the three models. Second, we introduce a new structural shock regarding R&D activities. In terms of the new R&D shock as well as standard DSGE shocks, we empirically classify the factors attributable to the long stagnation by calculating the variance and historical decompositions of the common trend and business cycle components.

We empirically find that, for Japan, the standard NK model with the TFP shock is superior to our models with the endogenous growth model, regardless of the presence or absence of nominal rigidities. However, limiting the model selection to the two endogenous growth models, the assumption of nominal rigidities of prices and wages is important to explain the data for over three decades. The R&D activity and investment shocks account for the larger portions of the business cycle components of the real GDP and investment. Furthermore, the common trends of the three DSGE models fluctuate with much greater volatility than those of both the HP and BP filters. Finally, we observe that the two deep declines in the R&D activities during the Asian financial crisis in 1998 and the Lehman Brothers' failure in 2008:Q3 produced not only big recessions in the business cycles but also stagnation of the economic growth.

## A Appendix

### A.1 Structural equations of the NK model with endogenous growth

#### A.1.1 R&D sector

##### Innovators

1. Innovation accumulation (R&D investment) from Eq.(1):

$$\frac{1}{a_t} = \frac{1 - \delta_z}{\mu_t} \frac{1}{a_{t-1}} + \phi_t i_{d,t}.$$

2. Efficiency of R&D investment from Eq.(2):

$$\phi_t = \frac{\chi_z}{a_{t-1} i_{d,t}^{1-\rho}}.$$

3. No-arbitrage condition for innovation from Eq.(3).

$$i_{d,t} = (1 - \delta_z) \mu_t^A \left( \frac{1}{a_t} - \frac{1 - \delta_z}{\mu_t^V a_{t-1}} \right) \mathbf{E}_t \Lambda_{t|t+1} j_{t+1}.$$

##### Adopters

1. Success probability from innovation to new product from Eq.(6):

$$\lambda_t = \lambda_0 i_{a,t}^{\omega_a} z_t^\lambda.$$

2. Technology progress rate from Eq.(7):

$$\mu_t^A = (\mu_t^V)^{\frac{\lambda_x}{1-\alpha}}.$$

3. Newly adopted innovation from Eq.(8):

$$\tilde{\Delta}_{a,t} = (1 - \delta_a) \left( \frac{1}{a_{t-1}} - 1 \right) \lambda_t.$$

4. FOC of adopted innovation from Eq.(9):

$$i_{a,t} = (1 - \delta_a) \omega_a \lambda_t \left( p_t^V - \frac{\mu_t^A}{\mu_t^V} \mathbf{E}_t \Lambda_{t|t+1} j_{t+1} \right).$$

5. Value of unadopted innovation from Eq.(10):

$$j_t = (1 - \delta_a) \left[ (1 - \omega_a) \lambda_t p_t^V + \{1 - (1 - \omega_a) \lambda_t\} \frac{\mu_t^A}{\mu_t^V} \mathbf{E}_t \Lambda_{t|t+1} j_{t+1} \right].$$

## A.1.2 Productive sector

### Wholesaler

1. Marginal cost of wholesaler from Eq.(17) and (32):

$$mc_t = (1 + \lambda^x) s_t.$$

2. Price version of New Keynesian Philips Curve (NKPC) 1 from Eq.(19):

$$\frac{\pi_t^p}{\pi_t} = (1 + \lambda^p z_t^p) \frac{k_t^p}{f_t^p}.$$

3. Price version of NKPC 2 from Eq. (20):

$$k_t^p = mc_t y_t + \xi_p \mathbf{E}_t \mu_t^A \Lambda_{t|t+1} \left( \frac{\pi_t^{\gamma_p} \pi^{1-\gamma_p}}{\pi_{t+1}} \right)^{-\frac{1+\lambda^p}{\lambda^p}} k_{t+1}^p.$$

4. Price version of NKPC 3 from Eq. (21):

$$f_t^p = y_t + \xi_p \mathbf{E}_t \mu_t^A \Lambda_{t|t+1} \left( \frac{\pi_t^{\gamma_p} \pi^{1-\gamma_p}}{\pi_{t+1}} \right)^{-\frac{1}{\lambda^p}} f_{t+1}^p.$$

5. Price version of NKPC 4 from Eq. (22):

$$1 = \xi_p \left( \frac{\pi_t^{\gamma_p} \pi^{1-\gamma_p}}{\pi_t} \right)^{-\frac{1}{\lambda^p}} + (1 - \xi_p) \left( \frac{\pi_t^o}{\pi_t} \right)^{-\frac{1}{\lambda^p}}.$$

### Intermediate Goods Firms

1. Implemented technology (goods variety) accumulation from Eq.(29) :

$$\mu_t^V = 1 - \delta_a + \left\{ 1 - S^a \left( \frac{\tilde{\Delta}_{a,t} \mu_{t-1}^V}{\tilde{\Delta}_{a,t-1} \mu^V} \right) \right\} \tilde{\Delta}_{a,t}.$$

2. Capital depreciation rate function from Eq.(27):

$$\delta(u_t^k) = \delta_k + b_k \frac{(u_t^k)^{1+\zeta_k}}{1 + \zeta_k}.$$

3. Marginal capital depreciation rate from Eq.(27) :

$$\delta'(u_t^k) = b_k (u_t^k)^{\zeta_k}.$$

4. Marginal productivity of capital stock from Eq.(33) :

$$r_t^k = \alpha s_t \frac{\mu_{t-1}^A y_t}{u_t^k k_{t-1}}.$$

5. Marginal productivity of labor from Eq.(34) :

$$w_t = (1 - \alpha) s_t \frac{y_t}{l_t}.$$

6. Realized technology price from Eq.(35) :

$$p_t^V = \gamma_t^a \left\{ 1 - S_a \left( \frac{\tilde{\Delta}_{a,t} \mu_{t-1}^V}{\tilde{\Delta}_{a,t-1} \mu^V} \right) - S'_a \left( \frac{\tilde{\Delta}_{a,t} \mu_{t-1}^V}{\tilde{\Delta}_{a,t-1} \mu^V} \right) \left( \frac{\tilde{\Delta}_{a,t} \mu_{t-1}^V}{\tilde{\Delta}_{a,t-1} \mu^V} \right) \right\} \\ + \mu^V \Lambda_{t|t+1} \gamma_{t+1}^a S'_a \left( \frac{\tilde{\Delta}_{a,t+1} \mu_t^V}{\tilde{\Delta}_{a,t} \mu^V} \right) \left( \frac{\tilde{\Delta}_{a,t+1} \mu_t^V}{\tilde{\Delta}_{a,t} \mu^V} \right)^2.$$

7. Goods variety adjustment cost function:

$$S_a \left( \frac{\tilde{\Delta}_{a,t} \mu_{t-1}^V}{\tilde{\Delta}_{a,t-1} \mu^V} \right) = \frac{1}{2} \frac{1}{\bar{\zeta}_a} \left( \frac{\tilde{\Delta}_{a,t} \mu_{t-1}^V}{\tilde{\Delta}_{a,t-1} \mu^V} - 1 \right)^2.$$

8. Derived Goods variety adjustment cost function:

$$S'_a \left( \frac{\tilde{\Delta}_{a,t} \mu_{t-1}^V}{\tilde{\Delta}_{a,t-1} \mu^V} \right) = \frac{1}{\zeta_a} \left( \frac{\tilde{\Delta}_{a,t} \mu_{t-1}^V}{\tilde{\Delta}_{a,t-1} \mu^V} - 1 \right).$$

9. Tobin's Q of goods variety Eq.(36):

$$\gamma_{a,t} \frac{\mu_t^V}{\mu_t^A} = \mathbf{E}_t \Lambda_{t|t+1} \{ (1 - \delta_a) \gamma_{a,t+1} + \lambda^x s_{t+1} y_{t+1} \}.$$

10. Production function from Eq.(26):

$$y_t = \left( \frac{u_t^k k_{t-1}}{\mu_{t-1}^A} \right)^\alpha l_t^{1-\alpha}.$$

### A.1.3 Households

1. Marginal utility of consumption from Eq.(40):

$$\lambda_t^c = \frac{\mu_{t-1}^A z_t^b}{\mu_{t-1}^A c_t - h c_{t-1}} - \beta h \mathbf{E}_t \frac{z_{t+1}^b}{\mu_t^A c_{t+1} - h c_t}.$$

2. Stochastic discount factor (SDF) from Eq.(41):

$$\mu_t^A \Lambda_{t|t+1} = \beta \mathbf{E}_t \frac{\lambda_{t+1}^c}{\lambda_t^c}.$$

3. Euler equation from Eq.(42):

$$\Lambda_{t|t+1} = \frac{\mathbf{E}_t \pi_{t+1}}{r_t^n}.$$

4. Fixed capital accumulation from Eq.(28):

$$k_t = \left\{ 1 - \delta \left( u_t^k \right) \right\} \frac{k_{t-1}}{\mu_{t-1}^A} + z_t^i \left\{ 1 - S \left( \frac{i_t}{i_{t-1}} \frac{\mu_{t-1}^A}{\mu^A} \right) \right\} i_t.$$

5. FOC with respect to capital utilization rate from Eq.(45):

$$r_t^k = q_t^k \delta' \left( u_t^k \right).$$

6. FOC with respect to fixed capital investment Eq.(43):

$$1 = z_t^i q_t^k \left\{ 1 - S \left( \frac{i_t}{i_{t-1}} \frac{\mu_{t-1}^A}{\mu^A} \right) - S' \left( \frac{i_t}{i_{t-1}} \frac{\mu_{t-1}^A}{\mu^A} \right) \left( \frac{i_t}{i_{t-1}} \frac{\mu_{t-1}^A}{\mu^A} \right) \right\} \\ + \mu^A \mathbf{E}_t z_{t+1}^i q_{t+1}^k \Lambda_{t|t+1} S' \left( \frac{i_{t+1}}{i_t} \frac{\mu_t^A}{\mu^A} \right) \left( \frac{i_t}{i_{t-1}} \frac{\mu_{t-1}^A}{\mu^A} \right)^2 .$$

7. Fixed capital investment adjustment cost function:

$$S \left( \frac{i_t}{i_{t-1}} \frac{\mu_{t-1}^A}{\mu^A} \right) = \frac{1}{2} \frac{1}{\bar{\zeta}} \left( \frac{i_t}{i_{t-1}} \frac{\mu_{t-1}^A}{\mu^A} - 1 \right)^2 .$$

8. Derived capital investment adjustment cost function:

$$S' \left( \frac{i_t}{i_{t-1}} \frac{\mu_{t-1}^A}{\mu^A} \right) = \frac{1}{\bar{\zeta}} \left( \frac{i_t}{i_{t-1}} \frac{\mu_{t-1}^A}{\mu^A} - 1 \right) .$$

9. FOC with respect to fixed capital Eq.(44):

$$q_t^k = \mathbf{E}_t \Lambda_{t|t+1} \left[ r_t^k u_t^k + q_{t+1}^k \left\{ 1 - \delta' \left( u_{t+1}^k \right) \right\} \right] .$$

10. Wage verion of New Keynesian Philips Curve (NKPC) 1 from Eq.(48) :

$$\frac{w_t^o}{w_t} = \left\{ (1 + \lambda^w z_t^w) \frac{k_t^w}{f_t^w} \right\}^{\frac{\lambda^w}{\lambda^w + \omega + \lambda^w \omega}} .$$

11. Wage verion of NKPC 2 from Eq.(49):

$$k_t^w = \gamma_l z_t^b z_t^l t^{1+\omega_l} + \beta \xi_w \mathbf{E}_t \left( \frac{\pi_t^{\gamma_w} \pi^{1-\gamma_w}}{\pi_{t+1}} \frac{w_t}{w_{t+1}} \frac{\mu^A}{\mu_{t+1}^A} \right)^{-\frac{1+\lambda^w}{\lambda^w} (1+\omega_l)} k_{t+1}^w .$$

12. Wage verion of NKPC 3 from Eq.(50):

$$f_t^w = \lambda_t^c l_t w_t + \beta \xi_w \mathbf{E}_t \left( \frac{\pi_t^{\gamma_w} \pi^{1-\gamma_w}}{\pi_{t+1}} \frac{w_t}{w_{t+1}} \frac{\mu_A}{\mu_{A,t}} \right)^{-\frac{1}{\lambda^w}} f_{t+1}^w .$$

13. Wage verion of NKPC 4 from Eq.(51) :

$$1 = \xi_w \left( \frac{\pi_{t-1}^{\gamma_w} \pi^{1-\gamma_w}}{\pi_t} \frac{w_{t-1}}{w_t} \frac{\mu^A}{\mu_{t-1}^A} \right)^{-\frac{1}{\lambda^w}} + (1 - \xi_w) \left( \frac{w_t^o}{w_t} \right)^{-\frac{1}{\lambda^w}} .$$



#### A.1.4 Miscellaneous

1. Market clearing from Eq.(52):

$$y_t = c_t + i_t + i_{a,t} \left( \frac{1}{a_{t-1}} - 1 \right) + i_{d,t} + \overline{g/y} z_t^g.$$

2. Monerary policy rule from Eq.(54):

$$\ln r_t^n = \phi_r \ln r_{t-1}^n + (1 - \phi_r) \left\{ \ln r^n + \phi_\pi \ln \left( \frac{\pi_t}{\pi} \right) + \phi_y \ln \left( \frac{y_t}{y} \right) \right\} + \varepsilon_t^r.$$

3. Observed output(GDP) Eq.(53):

$$\tilde{Y}_t = c_t + i_t + \overline{g/y} z_t^g.$$

#### A.1.5 Structural shocks with AR (1) processes

1. Preference shock:

$$\ln z_t^b = \rho_b \ln z_{t-1}^b + \varepsilon_t^b.$$

2. Labor disutility shock:

$$\ln z_t^l = \rho_l \ln z_{t-1}^l + \varepsilon_t^l.$$

3. Physical investment efficiency shock:

$$\ln z_t^i = \rho_i \ln z_{t-1}^i + \varepsilon_t^i.$$

4. Exogenous Spending shock:

$$\ln z_t^g = \rho_g \ln z_{t-1}^g + \varepsilon_t^g.$$

5. R&D efficiency shock:

$$\ln z_t^\lambda = \rho_\lambda \ln z_{t-1}^\lambda + \varepsilon_t^\lambda.$$

6. Price Markup Shock:

$$\ln z_t^p = \rho_p \ln z_{t-1}^p + \varepsilon_t^p.$$

7. Wage Markup Shock:

$$\ln z_t^w = \rho_w \ln z_{t-1}^w + \varepsilon_t^w.$$

## A.2 Structural equations of the standard NK model

### A.2.1 Productive sector

#### Intemediate Goods Firms

1. Production function:

$$y_t = \left( \frac{u_t^k k_{t-1}}{\mu_t^z} \right)^\alpha l_t^{1-\alpha}.$$

2. Stochastic trend from Eq.(55):

$$\mu_t^z \equiv \mu^z z_t^z.$$

3. Marginal productivity of capital stock:

$$r_t^k = \alpha m c_t \frac{\mu_t^z y_t}{u_t^k k_{t-1}}.$$

4. Marginal productivity of labor :

$$w_t = (1 - \alpha) m c_t \frac{y_t}{l_t}.$$

5. Price version of New Keynesian Philips Curve (NKPC) 1:

$$\frac{\pi_t^o}{\pi_t} = (1 + \lambda^p z_t^p) \frac{k_t^p}{f_t^p}.$$

6. Price version of NKPC 2 :

$$k_t^p = m c_t y_t + \xi_p \mathbf{E}_t \mu_t^z \Lambda_{t|t+1} \left( \frac{\pi_t^{\gamma_p} \pi^{1-\gamma_p}}{\pi_{t+1}} \right)^{-\frac{1+\lambda^p}{\lambda^p}} k_{t+1}^p.$$

7. Price version of NKPC 3 :

$$f_t^p = y_t + \xi_p \mathbf{E}_t \mu_t^z \Lambda_{t|t+1} \left( \frac{\pi_t^{\gamma_p} \pi^{1-\gamma_p}}{\pi_{t+1}} \right)^{-\frac{1}{\lambda^p}} f_{t+1}^p.$$

8. Price version of NKPC 4 :

$$1 = \xi_p \left( \frac{\pi_{t-1}^{\gamma_p} \pi^{1-\gamma_p}}{\pi_t} \right)^{-\frac{1}{\lambda^p}} + (1 - \xi_p) \left( \frac{\pi_t^o}{\pi_t} \right)^{-\frac{1}{\lambda^p}}.$$

## A.2.2 Household sector

1. Marginal utility of consumption :

$$\lambda_t^c = \frac{\mu_t^z z_t^b}{\mu_t^z c_t - h c_{t-1}} - \beta h \mathbf{E}_t \frac{z_{t+1}^b}{\mu_{t+1}^z c_{t+1} - h c_t}.$$

Stochastic discount factor (SDF):

$$\Lambda_{t|t+1} = \beta \mathbf{E}_t \frac{\lambda_{t+1}^c}{\mu_{t+1}^z \lambda_t^c}.$$

2. Euler equation :

$$\Lambda_{t|t+1} = \frac{\mathbf{E}_t \pi_{t+1}}{r_t^n}.$$

3. Fixed capital accumulation :

$$k_t = \left\{ 1 - \delta \left( u_t^k \right) \right\} \frac{k_{t-1}}{\mu_t^z} + z_t^i \left\{ 1 - S \left( z_t^z \frac{i_t}{i_{t-1}} \right) \right\} i_t.$$

4. Capital depreciation rate function :

$$\delta \left( u_t^k \right) = \delta_k + b_k \frac{\left( u_t^k \right)^{1+\zeta_k}}{1 + \zeta_k}.$$

5. Marginal capital depreciation rate :

$$\delta' \left( u_t^k \right) = b_k \left( u_t^k \right)^{\zeta_k}.$$

6. FOC with respect to capital utilization rate :

$$r_t^k = q_t^k \delta' \left( u_t^k \right).$$

7. FOC with respect to fixed capital investment :

$$1 = z_t^i q_t^k \left\{ 1 - S \left( z_t^z \frac{i_t}{i_{t-1}} \right) - S' \left( z_t^z \frac{i_t}{i_{t-1}} \right) \left( z_t^z \frac{i_t}{i_{t-1}} \right) \right\} \\ + \mu^z \mathbf{E}_t z_{t+1}^i q_{t+1}^k \Lambda_{t|t+1} S' \left( z_{t+1}^z \frac{i_{t+1}}{i_t} \right) \left( z_{t+1}^z \frac{i_{t+1}}{i_t} \right)^2.$$

8. Fixed capital investment adjustment cost function:

$$S \left( z_t^z \frac{i_t}{i_{t-1}} \right) = \frac{1}{2} \frac{1}{\zeta} \left( z_t^z \frac{i_t}{i_{t-1}} - 1 \right)^2.$$

9. Derived capital investment adjustment cost function:

$$S' \left( z_t^z \frac{i_t}{i_{t-1}} \right) = \frac{1}{\zeta} \left( z_t^z \frac{i_t}{i_{t-1}} - 1 \right).$$

10. FOC with respect to fixed capital:

$$q_t^k = \mathbf{E}_t \Lambda_{t|t+1} \left[ r_t^k u_t^k + q_{t+1}^k \left\{ 1 - \delta' \left( u_{t+1}^k \right) \right\} \right].$$

11. Wage verion of New Keynesian Philips Curve (NKPC) 1:

$$\frac{w_t^o}{w_t} = \left\{ \left( 1 + \lambda^w z_t^w \right) \frac{k_t^w}{f_t^w} \right\}^{\frac{\lambda^w}{\lambda^w + \omega + \lambda^w \omega}}.$$

12. Wage verion of NKPC 2:

$$k_t^w = \gamma_l z_t^b z_t^l l_t^{1+\omega_l} + \beta \xi_w \mathbf{E}_t \left( \frac{\pi_t^{\gamma_w} \pi^{1-\gamma_w}}{\pi_{t+1}} \frac{w_t}{w_{t+1} z_{t+1}^z} \right)^{-\frac{1+\lambda^w}{\lambda^w} (1+\omega_l)} k_{t+1}^w.$$

13. Wage verion of NKPC 3:

$$f_t^w = \lambda_t^c l_t w_t + \beta \xi_w \mathbf{E}_t \left( \frac{\pi_t^{\gamma_w} \pi^{1-\gamma_w}}{\pi_{t+1}} \frac{w_t}{w_{t+1} z_{t+1}^z} \right)^{-\frac{1}{\lambda^w}} f_{t+1}^w.$$

14. Wage verion of NKPC 4:

$$1 = \xi_w \left( \frac{\pi_{t-1}^{\gamma_w} \pi^{1-\gamma_w}}{\pi_t} \frac{w_{t-1}}{w_t z_t^z} \right)^{-\frac{1}{\lambda^w}} + (1 - \xi_w) \left( \frac{w_t^o}{w_t} \right)^{-\frac{1}{\lambda^w}}.$$

### A.2.3 Miscellaneous

1. Market clearing:

$$y_t = c_t + i_t + \overline{g/y} z_t^g.$$

2. Monerary policy rule:

$$\ln r_t^n = \phi_r \ln r_{t-1}^n + (1 - \phi_r) \left\{ \ln r^n + \phi_\pi \ln \left( \frac{\pi_t}{\pi} \right) + \phi_y \ln \left( \frac{y_t}{y} \right) \right\} + \varepsilon_t^r.$$

### A.2.4 Structural shocks with AR (1) processes

1. Preference shock:

$$\ln z_t^b = \rho_b \ln z_{t-1}^b + \varepsilon_t^b.$$

2. Labor disutility shock:

$$\ln z_t^l = \rho_l \ln z_{t-1}^l + \varepsilon_t^l.$$

3. Physical investment efficiency shock:

$$\ln z_t^i = \rho_i \ln z_{t-1}^i + \varepsilon_t^i.$$

4. Exogenous spending shock:

$$\ln z_t^g = \rho_g \ln z_{t-1}^g + \varepsilon_t^g.$$

5. Technology progress rate shock:

$$\ln z_t^z = \rho_z \ln z_{t-1}^z + \varepsilon_t^z.$$

6. Price markup shock:

$$\ln z_t^p = \rho_p \ln z_{t-1}^p + \varepsilon_t^p.$$

7. Wage markup shock:

$$\ln z_t^w = \rho_w \ln z_{t-1}^w + \varepsilon_t^w.$$

### A.3 Derivation of the Production Function, Eq.(37)

In this section, we derive the production function described in Section 3. According to Section 3, we know that the price equation of final goods is given as

$$P_t^x = \left( \int_0^{V_{t-1}} P_{h,t}^x^{-\frac{1}{\lambda_x}} dh \right)^{-\lambda_x}, \quad (64)$$

and that the marginal cost of intermediate goods is given as

$$P_t(1 + \lambda_x) S_t \varphi_t' = P_{h,t}^x. \quad (65)$$

Using Eq.(64) and Eq.(65), we obtain

$$P_t^x = \left\{ \int_0^{V_{t-1}} \left\{ P_t(1 + \lambda_x) S_t \varphi_t' \right\}^{-\frac{1}{\lambda_x}} dh \right\}^{-\lambda_x}.$$

Here, since the term  $P_t (1 + \lambda_x) S_t \varphi_t'$  in the RHS does not depend on index  $h$ , the term can be moved in the front of integration  $\int_0^{V_{t-1}}$ . Hence, we get as follows.

$$\begin{aligned} \Leftrightarrow P_t^x &= P_t (1 + \lambda_x) S_t \varphi_t' \left\{ \int_0^{V_{t-1}} dh \right\}^{-\lambda_x}, \\ \Leftrightarrow P_t^x &= P_t (1 + \lambda_x) S_t \varphi_t' V_{t-1}^{-\lambda_x}. \end{aligned} \quad (66)$$

Using Eq.(66) and Eq.(65), we obtain

$$\begin{aligned} \frac{P_{h,t}^x}{P_t^x} &= \frac{P_t (1 + \lambda_x) S_t \varphi_t'}{P_t (1 + \lambda_x) S_t \varphi_t' V_{t-1}^{-\lambda_x}} = V_{t-1}^{\lambda_x}, \\ \frac{P_{f,t}^x}{P_t^x} &= V_{t-1}^{\lambda_x}. \end{aligned} \quad (67)$$

Hence, by substituting Eq.(67) into  $(u_t^k K_{t-1})^\alpha l_t^{1-\alpha} = X_t \int_0^{V_{t-1}} \left( \frac{P_{h,t}^x}{P_t^x} \right)^{-\frac{1+\lambda_x}{\lambda_x}} dh$ , we obtain

$$\begin{aligned} (u_t^k K_{t-1})^\alpha l_t^{1-\alpha} &= X_t \int_0^{V_{t-1}} \left( V_{t-1}^{\lambda_x} \right)^{-\frac{1+\lambda_x}{\lambda_x}} dh, \\ (u_t^k K_{t-1})^\alpha l_t^{1-\alpha} &= X_t V_{t-1}^{-1-\lambda_x} \int_0^{V_{t-1}} dh, \\ X_t &= V_{t-1}^{\lambda_x} (u_t^k K_{t-1})^\alpha l_t^{1-\alpha}, \\ X_t &= (u_t^k K_{t-1})^\alpha \left( V_{t-1}^{\frac{\lambda_x}{1-\alpha}} l_t \right)^{1-\alpha} \end{aligned} \quad (68)$$

Here, we set Harod-type neutral technology level  $A_t$  as  $A_t \equiv V_t^{\frac{\lambda_x}{1-\alpha}}$ , then we obtain our objective equation as

$$X_t = (u_t^k K_{t-1})^\alpha (A_{t-1} l_t)^{1-\alpha}.$$

## References

- Adolfson, M., S. Laséen, J. Lindé, and M. Villani (2007) ‘Bayesian estimation of an open economy dsge model with incomplete pass-through.’ *Journal of International Economics* 72(2), 481–511
- Altig, David, Lawrence J. Christiano, Martin Eichenbaum, and Jesper Linde (2011) ‘Firm-specific capital, nominal rigidities and the business cycle.’ *Review of Economic Dynamics* 14(2), 225–247. Bank of Japan Working Paper Series
- Christiano, Lawrence J., and Terry J. Fitzgerald (2003) ‘The band pass filter.’ *International Economic Review* 44(2), 435–465
- Christiano, Lawrence J., Mathias Trabandt, and Karl Walentin (2011) ‘Introducing financial frictions and unemployment into a small open economy model.’ *Journal of Economic Dynamics and Control* 35(12), 1999–2041
- Comin, Diego, and Mark Gertler (2006) ‘Medium-term business cycles.’ *The American Economic Review* 96(3), 523–551
- Erceg, C. J., D. W. Henderson, and A. T. Levin (2000) ‘Optimal monetary policy with staggered wage and price contracts.’ *Journal of Monetary Economics* 46(2), 281–313
- Gelman, Andrew, and Donald B. Rubin (1992) ‘Inference from iterative simulation using multiple sequences.’ *Statistical Science* 7(4), 457–472
- Geweke, John (1999) ‘Using simulation methods for bayesian econometric models: Inference, development, and communication.’ *Econometric Reviews* 18(1), 1–73
- Guerron-Quintana, P., and R. Jinnai (2015) ‘Liquidity shocks and asset prices.’ mimeo
- Guerron-Quintana, P., T. Hirano, and R. Jinnai (2016) ‘Bubbles and trends.’ mimeo
- Hayashi, Fumio, and Edward C. Prescott (2002) ‘The 1990s in japan: A lost decade.’ *Review of Economic Dynamics* 5(1), 206–235
- Hirose, Yasuo, and Atsushi Inoue (2016) ‘The zero lower bound and parameter bias in an estimated dsge model.’ *Journal of Applied Econometrics* 31(4), 630–651
- Iiboshi, Hirokuni, Tatsuyoshi Matsumae, Ryoichi Namba, and Shin-Ichi Nishiyama (2015) ‘Estimating a dsge model for japan in a data-rich environment.’ *Journal of the Japanese and International Economies* 36, 25–55
- Ikeda, Daisuke, and Takushi Kurozumi (2014) ‘Post-crisis slow recovery and monetary policy.’ IMES Discussion Paper Series, no. 2014-E-16

- Ikeda, Daisuke, and Yasuko Morita (2016) 'The effects of barriers to technology adoption on japanese prewar and postwar economic growth.' IMES Discussion Paper Series, no. 2016-E-1
- Justiniano, Alejandro, Giorgio E. Primiceri, and Andrea Tambalotti (2010) 'Investment shocks and business cycles.' *Journal of Monetary Economics* 57(2), 132–145
- Kaihatsu, Sohei, and Takushi Kurozumi (2014) 'Sources of business fluctuations: Financial or technology shocks?' *Review of Economic Dynamics* 17(2), 224–242
- King, Robert G., Charles I. Plosser, James H. Stock, and Mark W. Watson (1991) 'Stochastic trends and economic fluctuations.' *The American Economic Review* 81(4), 819–840
- Romer, Paul M (1990) 'Endogenous technological change.' *Journal of Political Economy* 98(5), 71–102
- Sugo, Tomohiro, and Kozo Ueda (2008) 'Estimating a dynamic stochastic general equilibrium model for japan.' *Journal of the Japanese and International Economies* 22(4), 476–502



Table 1: Observable Variables

Variable	Definition	Unit	Source
$Y_t^{data}$	Real GDP* <sup>1</sup>	a billion yen	SNA
$C_t^{data}$	Real private consumption	a billion yen	SNA
$I_t^{data}$	Real private investment	a billion yen	SNA
$W_t^{data}$	Real wage indices	2010 average = 100	MHLW* <sup>2</sup>
$l_t^{data}$	Worked hour indices	2010 average = 100	MHLW* <sup>3</sup>
$R_t^{n,data}$	Secured overnight call rate	%	Bank of Japan
$P_t^{data}$	GDP deflator	2005 year = 100	SNA
$N_t^{data}$	Labor force	a thousand	Statistic Bureau, MIC

Notes:

MHLW: Ministry of Health, Labor and Welfare, MIC: Ministry of Internal Affairs and Communications

\*1: Including net export and government spending

\*2: Monthly Labor Survey, real wage indices (2010 average = 100)

\*3: Monthly Labor Survey, seasonally adjusted worked hour indices (2010 average = 100, S.A.)

Table 2: Calibrated Parameters

	Parameter	Value	Reference
$h$	Habit formation	0.8500	-
$\beta$	Subjective discount factor	0.9975	Ikeda & Kurozumi(2014)
$\delta_z$	Obsolescence rate of innovator * <sup>1</sup>	0.0250	-
$\delta_{ss}$	Steady state of capital depreciation rate	0.0025	-
$\lambda_x$	Markup rate of wholesaler	0.3000	-
$\lambda_a$	Markup rate of final goods firm	0.4500	-
$\lambda_p$	Markup rate of intermediate goods firm	0.3000	-
$\lambda_w$	Wage markup rate	0.1500	-
$\mu_a, \mu_z$	Steady state technology progress rate	1.0050	Approximation of data-mean
$\frac{g}{y}$	Exogenous demand share of output	0.2500	Approximation of data-mean
$\alpha$	Capital share	0.4000	-

Notes:

\*1: We assume the obsolescence rate is common in innovator, adopter and wholesaler, so ideas and goods varieties depreciate at the same speed.

Table 3: Prior and Posterior of Parameters

	Definitions	Prior			Posterior								
		mean	St. D.	dist.	Benchmark		w/o Nominal Rigidity		Standard NK model				
					means	90% interval*1	means	90% interval*1	means	90% interval*1			
$h$	habit formation	0.85	0.1	beta	0.958	0.936	0.983	0.947	0.931	0.964	0.957	0.936	0.978
$\omega_l$	inverse of elasticity of labor supply	2	0.75	gamm	1.638	0.817	2.432	37.164	32.018	42.279	2.198	1.111	3.215
$\zeta_k$	Inverse of elasticity of utilization rate adjustment cost	1	0.1	gamm	0.952	0.804	1.104	0.783	0.649	0.917	1.147	0.983	1.308
$\frac{1}{\zeta}$	Coefficient of investment adjustment cost	4	1.5	gamm	3.412	2.206	4.607	5.953	3.756	8.178	8.314	5.073	11.506
$\frac{1}{\zeta_a}$	Coefficient of variety adjustment cost	3	1.5	gamm	1.373	0.333	2.381	1.108	0.263	1.944	N.A	N.A	N.A
$\lambda_{ss}$	Steady-state probability of technology adoption	0.025	0.003	beta	0.025	0.020	0.030	0.024	0.019	0.028	N.A	N.A	N.A
$\omega_a$	Elasticity of technology adoption probability	0.5	0.01	gamm	0.487	0.471	0.503	0.517	0.499	0.534	N.A	N.A	N.A
$\rho$	Elasticity of R & D productivity	0.6	0.1	beta	0.173	0.105	0.237	0.255	0.154	0.354	N.A	N.A	N.A
$\bar{l}$	Normalized steady-state labor	1.115	0.005	gamm	1.114	1.106	1.122	1.113	1.105	1.121	1.114	1.105	1.121
$\pi_{ss}$	Steady-state inflation rate	1.003	0.001	gamm	1.003	1.001	1.004	1.003	1.001	1.004	1.003	1.001	1.004
$\gamma_p$	Intermediate-good price indexation	0.4	0.15	beta	0.089	0.022	0.155	N.A	N.A	N.A	0.113	0.027	0.198
$\gamma_w$	Wage indexation	0.4	0.15	beta	0.292	0.091	0.490	N.A	N.A	N.A	0.305	0.086	0.502
$\xi_p$	Intermediate-good price stickiness	0.375	0.1	beta	0.957	0.940	0.974	0	N.A	N.A	0.961	0.949	0.975
$\xi_w$	Wage stickiness	0.375	0.1	beta	0.658	0.596	0.726	0	N.A	N.A	0.648	0.580	0.718
$\phi_r$	Monetary policy rate smoothing	0.8	0.1	beta	0.726	0.608	0.847	0.110	0.063	0.157	0.860	0.768	0.948
$\phi_\pi$	Monetary policy response to inflatio	1.7	0.1	gamm	1.619	1.465	1.779	2.601	2.416	2.790	1.609	1.451	1.762
$\phi_y$	Monetary policy response to output	0.125	0.05	gamm	0.183	0.106	0.259	0.107	0.049	0.164	0.286	0.166	0.407

Table 3: (continued)

Definitions	Prior			Posterior								
	Mean	St. Dev.	Dist.	Benchmark		w/o Nominal Rigidity		Standard NK model				
				means	90% interval*1	means	90% interval*1	means	90% interval*1			
$\rho_b$	0.5	0.1	beta	0.267	0.168	0.362	0.365	0.264	0.467	0.293	0.182	0.402
$\rho_l$	0.5	0.1	beta	0.513	0.444	0.580	0.934	0.909	0.959	0.636	0.552	0.722
$\rho_i$	0.5	0.2	beta	0.439	0.319	0.561	0.227	0.120	0.329	0.301	0.184	0.421
$\rho_g$	0.5	0.2	beta	0.942	0.903	0.985	0.946	0.915	0.977	0.931	0.890	0.975
$\rho_\lambda$	0.5	0.1	beta	0.920	0.892	0.947	0.967	0.952	0.983	N.A	N.A	N.A
$\rho_p$	0.5	0.1	beta	0.607	0.455	0.765	0.925	0.905	0.944	0.606	0.457	0.765
$\rho_w$	0.5	0.1	beta	0.502	0.334	0.658	0.510	0.352	0.677	0.500	0.335	0.666
$\sigma_b$	0.5	Inf	invg	0.258	0.136	0.379	0.209	0.147	0.269	0.235	0.136	0.333
$\sigma_l$	2	0.1	invg	2.021	1.860	2.178	1.569	1.469	1.669	2.006	1.839	2.170
$\sigma_i$	0.5	Inf	invg	0.138	0.093	0.178	0.235	0.156	0.311	0.325	0.196	0.453
$\sigma_g$	0.5	Inf	invg	0.044	0.039	0.048	0.043	0.039	0.048	0.052	0.047	0.058
$\sigma_r$	0.5	Inf	invg	0.034	0.031	0.038	0.036	0.032	0.040	0.034	0.031	0.038
$\sigma_\lambda$	0.5	Inf	invg	0.044	0.039	0.049	0.038	0.034	0.042	N.A	N.A	N.A
$\sigma_p$	0.5	0.2	invg	0.599	0.372	0.821	0.184	0.164	0.204	0.615	0.393	0.827
$\sigma_w$	0.5	0.2	invg	0.495	0.252	0.738	0.452	0.249	0.653	0.494	0.247	0.746
$\rho_z$	0.5	0.1	beta	N.A	N.A	N.A	N.A	N.A	N.A	0.281	0.175	0.374
$\epsilon^z$	0.5	Inf	IG	N.A	N.A	N.A	N.A	N.A	N.A	0.043	0.039	0.048

Notes:

We estimated the model parameters during the sample period: 1980:Q2 through 2013:Q4, using MCMC simulation, in which we generated 3 chains of 125,000 draws from the posterior distribution of parameters by the Metropolis-Hastings algorithm and discarded the first 20% of each chain as burn-in iterations.

To diagnose the convergence, we calculated the Gelman and Rubin (1992)'s diagnostic statistics and confirmed that the statistics of all parameters were below 1.10 which is regarded as a crude measure of convergence.

\*1: 90% Highest posterior density interval.

Table 4: Model Selection

Model	Marginal Likelihood
Benchmark	2279.481
w/o nominal rigidity	2099.340
Standard NK model	2302.911

Notes: The marginal likelihoods of the three models are calculated from the posterior density of parameters and sampled by using the harmonic mean method proposed by Geweke (1999).

Table 5: Standard Deviations and Correlations of Business Cycle Components

(a) Standard Deviations of Business Cycle Components

	Benchmark	w/o Nominal rigidity	Standard NK	HP filter <sup>*1,2</sup>	BP filter <sup>*1,3</sup>
Output	0.0479	0.0453	0.0435	0.0143	0.0126
Cons.	0.0402	0.0331	0.0324	0.0115	0.0092
Inv.	0.1177	0.1194	0.1157	0.0655	0.0584

(b) Correlation of Output

Model	Benchmark	w/o Nominal rigidity	Standard NK	HP	BP
Benchmark	1				
w/o Nominal rigidity	0.981	1			
Standard NK	0.856	0.857	1		
HP	0.411	0.427	0.319	1	
BP	0.377	0.386	0.365	0.879	1

(c) Correlation of Consumption

Model	Benchmark	w/o Nominal rigidity	Standard NK	HP	BP
Benchmark	1				
w/o Nominal rigidity	0.978	1			
Standard NK	0.591	0.536	1		
HP	0.425	0.462	0.003	1	
BP	0.336	0.367	0.104	0.797	1

(d) Correlation of Investment

Model	Benchmark	w/o Nominal rigidity	Standard NK	HP	BP
Benchmark	1				
w/o Nominal rigidity	0.996	1			
Standard NK	0.960	0.965	1		
HP	0.767	0.758	0.756	1	
BP	0.549	0.543	0.582	0.882	1

Note: Panel (a) shows the standard deviations of the business cycle components derived from the five methods. The series extracted from the DSGE models is standardized to 1980:Q2 = 100. ‘Benchmark’ and ‘w/o Nominal rigidity’ represent our DSGE models with endogenous growth and ‘standard NK model’ represents our DSGE model without endogenous growth. ‘HP’ and ‘BP’ denote Hodrick-Prescott filter and Christiano-Fitzgerald bandpass filter, respectively. From Panel (b) to Panel (d), correlation coefficients among each business cycle component are described.

\*1 Both HP and BP filters are implemented for level of observations, but not the first difference of observations.

\*2 Parameter  $\lambda$  (the degree of smoothness) of the HP filter is set to 1,600.

\*3 The BP filter extracts the business cycle components regarding frequencies from 6 to 32 quarters.

Table 6: Variance Decomposition for Longer Cycles

(a) Benchmark Model: longer cycles ( $T \geq 32Q$ )									
Observation	Pref. $\epsilon^b$	Labor $\epsilon^l$	Inv. $\epsilon^i$	Exg. $\epsilon^g$	MP $\epsilon^R$	Price $\epsilon^P$	Wage $\epsilon^w$	Suc. Prob $\epsilon^\lambda$	TFP $\epsilon^z$
$\Delta Y_t^{obs}$	0.0	0.0	0.5	1.2	69.0	0.0	0.0	29.4	n.a.
$\Delta C_t^{obs}$	5.5	0.0	0.2	15.5	36.4	0.0	0.0	42.2	n.a.
$\Delta I_t^{obs}$	0.1	0.0	0.7	5.3	77.5	0.0	0.0	16.4	n.a.

(b) w/o Nominal Rigidity: longer cycles ( $T \geq 32Q$ )									
Observation	Pref. $\epsilon^b$	Labor $\epsilon^l$	Inv. $\epsilon^i$	Exg. $\epsilon^g$	MP $\epsilon^R$	Price $\epsilon^P$	Wage $\epsilon^w$	Suc. Prob $\epsilon^\lambda$	TFP $\epsilon^z$
$\Delta Y_t^{obs}$	0.2	0.2	0.2	11.2	0.0	1.8	0.0	86.5	n.a.
$\Delta C_t^{obs}$	7.2	0.1	0.3	34.4	0.0	0.4	0.0	57.7	n.a.
$\Delta I_t^{obs}$	0.9	0.3	0.7	38.0	0.0	3.4	0.0	56.8	n.a.

(c) Standard NK: longer cycles ( $T \geq 32Q$ )									
Observation	Pref. $\epsilon^b$	Labor $\epsilon^l$	Inv. $\epsilon^i$	Exg. $\epsilon^g$	MP $\epsilon^R$	Price $\epsilon^P$	Wage $\epsilon^w$	Suc. Prob $\epsilon^\lambda$	TFP $\epsilon^z$
$\Delta Y_t^{obs}$	0.0	0.0	0.0	0.3	32.5	0.0	0.0	n.a.	67.2
$\Delta C_t^{obs}$	0.5	0.0	0.0	0.8	4.7	0.0	0.0	n.a.	94.1
$\Delta I_t^{obs}$	0.0	0.0	0.0	1.1	55.8	0.0	0.0	n.a.	43.0

Notes: Following Justiniano et al. (2011), we conduct unconditional variance decomposition regarding longer cycles containing trend components at frequencies over 32 quarters.

Pref., Inv., Exp, MP, Price, Wage and Suc. Prob stand for the preference shock, investment efficient shock, exogenous expenditure shock, monetary policy shock, price markup shock, wage markup shock, and success probability shock, respectively.

Table 7: Variance Decomposition for Standard Business Cycles

(a) Benchmark Model: standard business cycles ( $6Q \leq T \leq 32Q$ )									
Observation	Pref. $\epsilon^b$	Labor $\epsilon^l$	Inv. $\epsilon^i$	Exg. $\epsilon^g$	MP $\epsilon^R$	Price $\epsilon^P$	Wage $\epsilon^w$	Suc. Prob $\epsilon^\lambda$	TFP $\epsilon^z$
$\Delta Y_t^{obs}$	0.1	0.0	1.8	7.2	87.4	0.0	0.0	3.6	n.a.
$\Delta C_t^{obs}$	48.4	0.0	0.0	7.0	32.3	0.0	0.0	12.2	n.a.
$\Delta I_t^{obs}$	0.0	0.0	2.3	1.7	94.2	0.0	0.0	1.8	n.a.

(b) w/o Nominal Rigidity: standard business cycles ( $6Q \leq T \leq 32Q$ )									
Observation	Pref. $\epsilon^b$	Labor $\epsilon^l$	Inv. $\epsilon^i$	Exg. $\epsilon^g$	MP $\epsilon^R$	Price $\epsilon^P$	Wage $\epsilon^w$	Suc. Prob $\epsilon^\lambda$	TFP $\epsilon^z$
$\Delta Y_t^{obs}$	0.9	0.1	3.0	80.5	0.0	1.5	0.0	14.0	n.a.
$\Delta C_t^{obs}$	67.9	0.1	0.1	20.8	0.0	0.1	0.0	11.1	n.a.
$\Delta I_t^{obs}$	1.5	0.4	11.8	58.9	0.0	4.0	0.0	23.5	n.a.

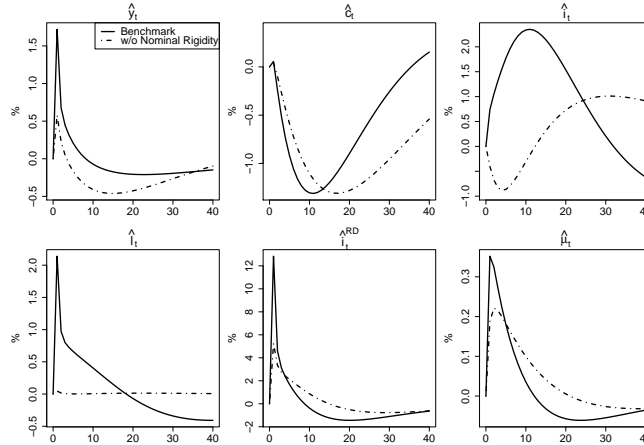
  

(c) Standard NK: standard business cycles ( $6Q \leq T \leq 32Q$ )									
Observation	Pref. $\epsilon^b$	Labor $\epsilon^l$	Inv. $\epsilon^i$	Exg. $\epsilon^g$	MP $\epsilon^R$	Price $\epsilon^P$	Wage $\epsilon^w$	Suc. Prob $\epsilon^\lambda$	TFP $\epsilon^z$
$\Delta Y_t^{obs}$	0.0	0.0	0.2	3.6	58.1	0.0	0.0	n.a.	38.1
$\Delta C_t^{obs}$	10.1	0.0	0.0	1.1	20.5	0.0	0.0	n.a.	68.4
$\Delta I_t^{obs}$	0.0	0.0	0.2	0.6	80.4	0.0	0.0	n.a.	18.8

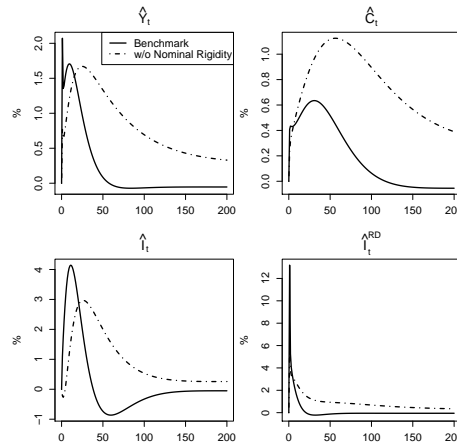
Notes: Following Justiniano et al. (2011), we conduct unconditional variance decomposition regarding standard business cycle frequencies from 6 to 32 quarters. Pref., Inv., Exp, MP, Price, Wage and Suc. Prob stand for the preference shock, investment efficient shock, exogenous expenditure shock, monetary policy shock, price markup shock, wage markup shock, and success probability shock, respectively.

Figure 1: Impulse Response

(a) Impulse Responses of the Detrended Variables



(b) Impulse Responses (Deviation from the Balanced Growth Path)



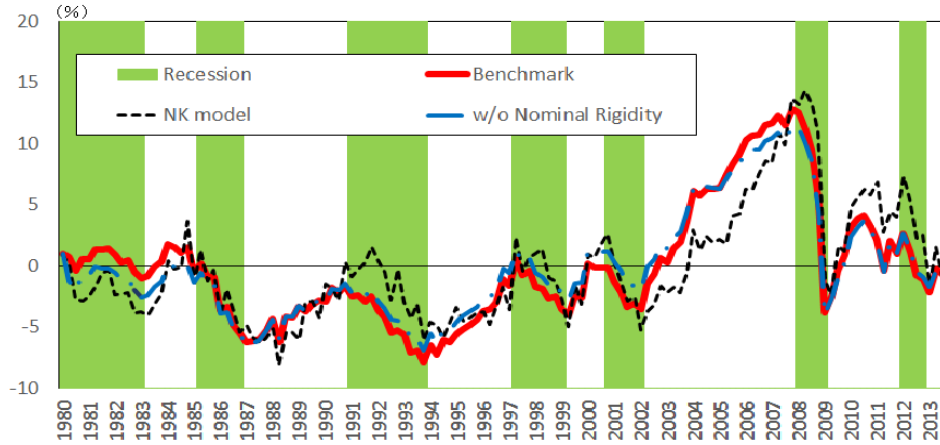
Notes: The impulse responses of endogenous variables to a one standard deviation success rate shock are drawn: the solid lines are those of the benchmark model and the dash-dotted lines are those of the model without nominal rigidity. All parameter are set to the posterior mean described in Table 3.

Panel (a) shows the impulse responses of the detrended variables, and Panel (b) shows that the deviation from the balanced growth path (trend of growth) when a success rate shock has occurred.

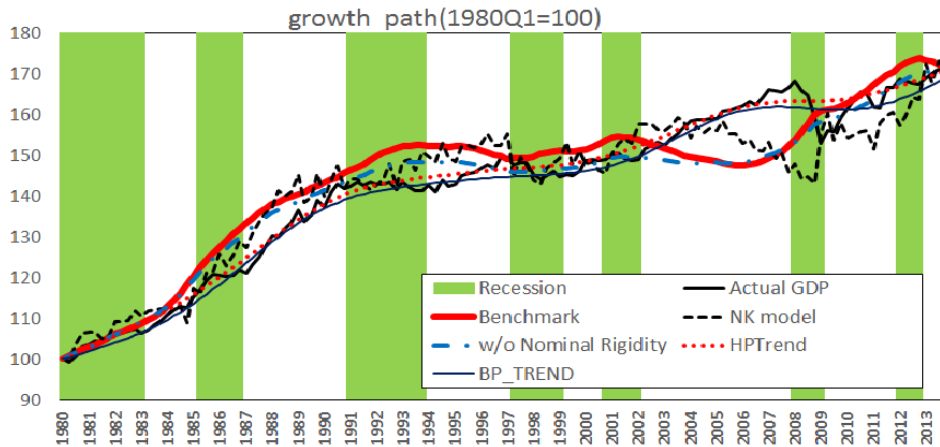


Figure 2: Business Cycle and Common Trend Components

(a) Business Cycle Component of Real GDP



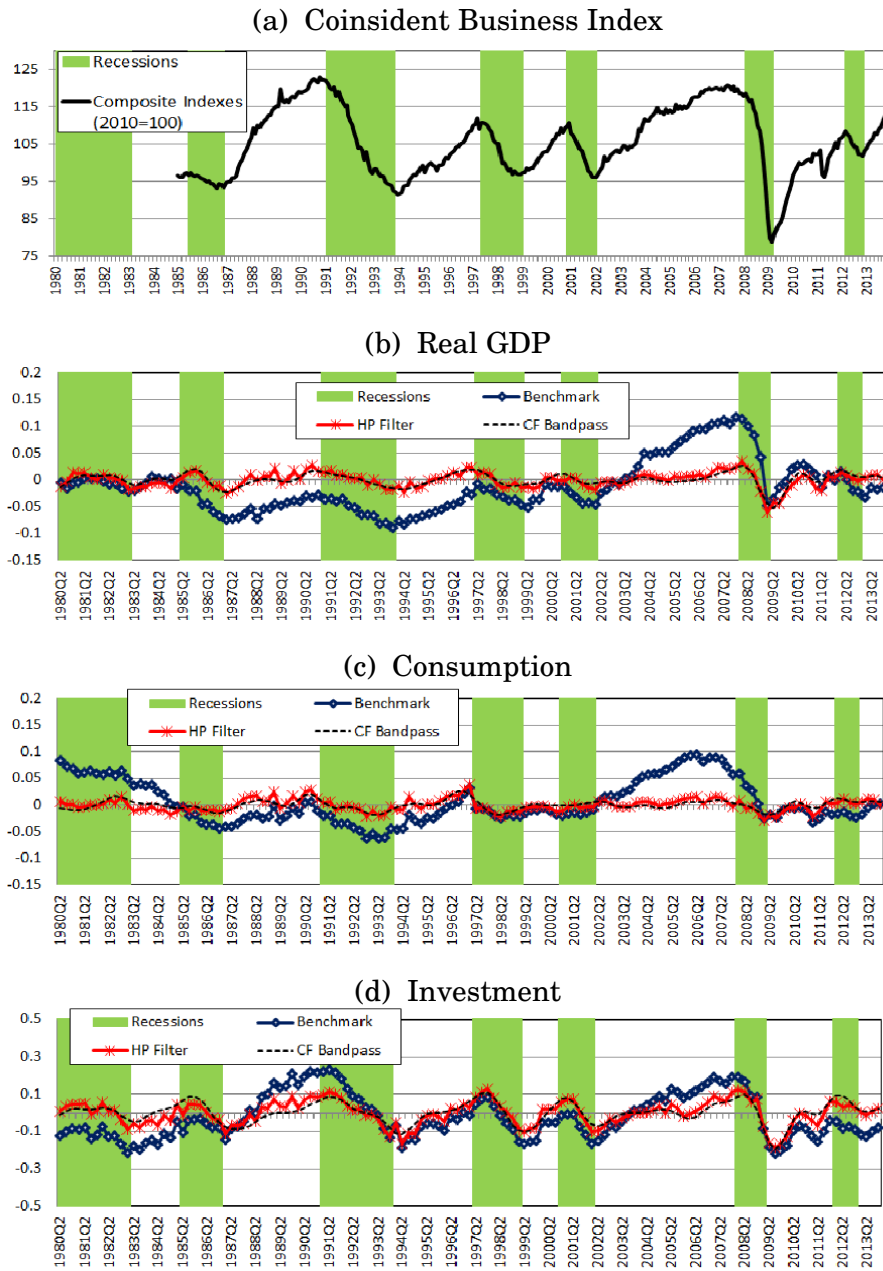
(b) Common Trend Component of Real GDP



Notes: Panel (a) shows the cycle components of output in the three DSGE models: the red solid line, the black dashed line and the blue line stand for the benchmark, the standard NK model and w/o nominal rigidity model, respectively. The green shaded areas denote recessions reported by the ESRI.

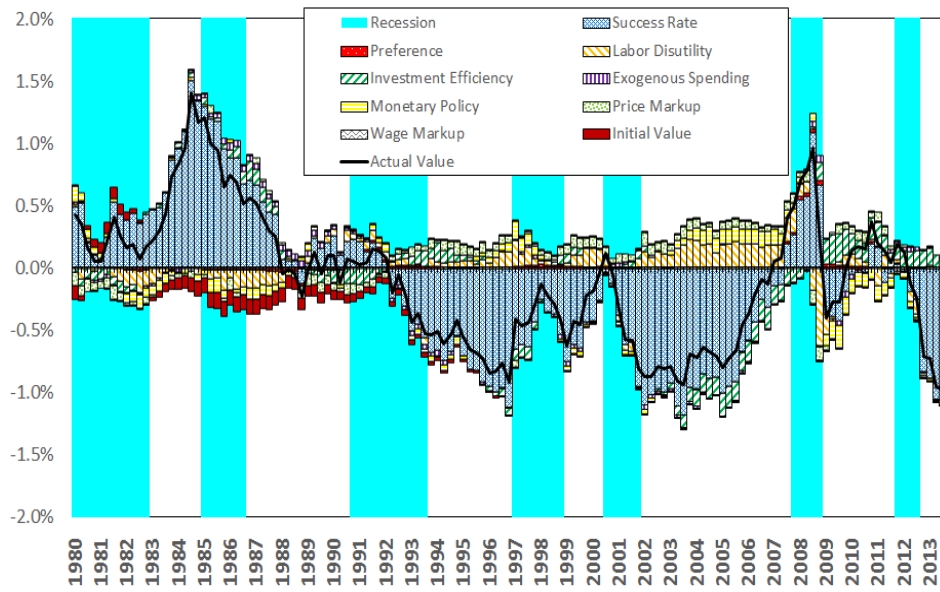
Panel (b) depicts the common trend components of real GDP in terms of the five approaches including two filtering approaches: the Hodrick-Prescott (HP) filter and band-pass (BP) filter in the setting described in the footnote in Table 5.

Figure 3: Coincident Business Index and Cycle Components



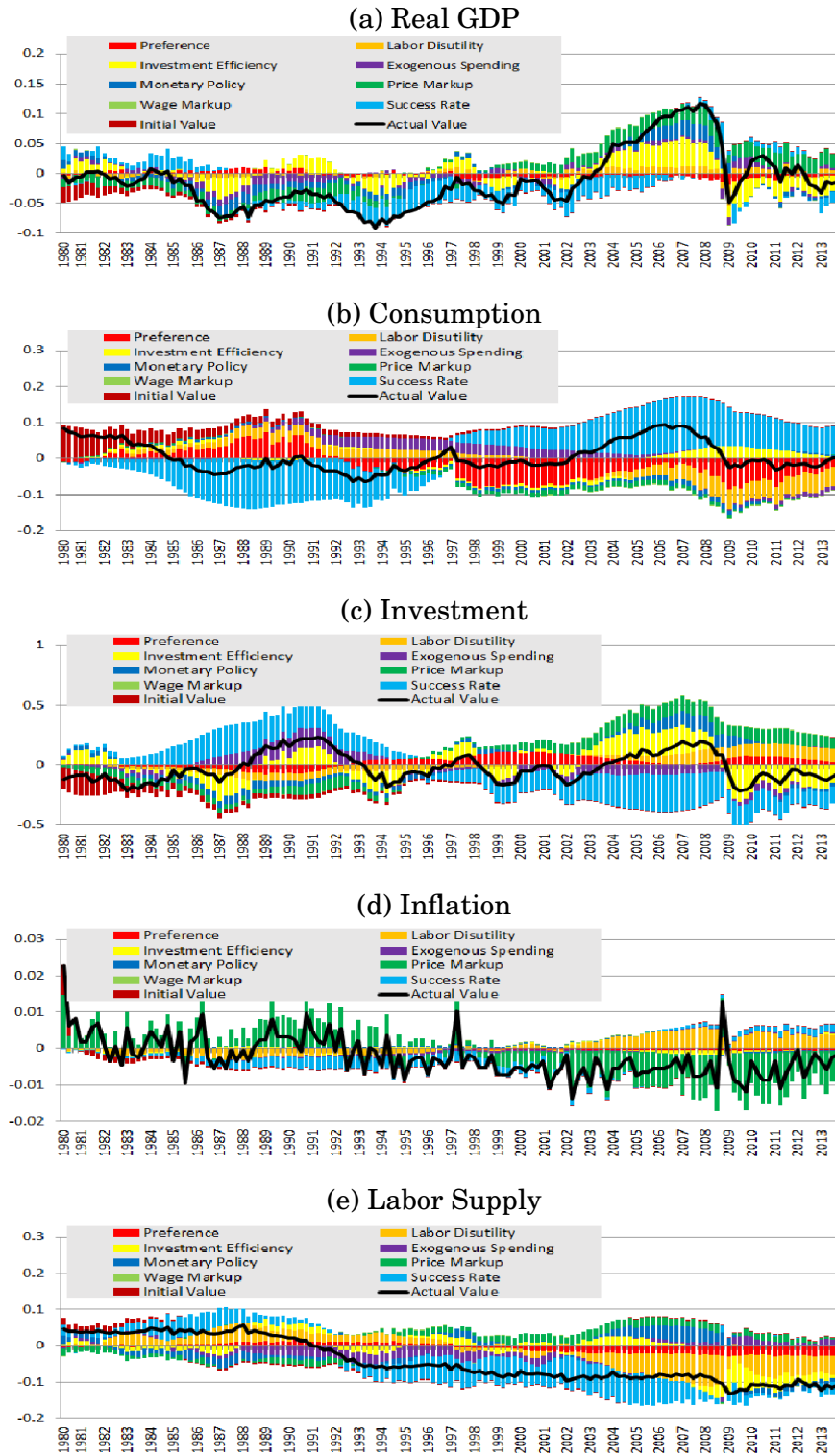
Notes: Panel (a) shows the coincident business cycle index (CI) reported by the ESRI. In Panel (b) through (d), the blue solid lines marked with diamond symbols represent the cycle components corresponding to the three observations of the benchmark, while the red solid lines with asterisk symbols and the black dashed lines are those extracted from the HP and BP filters, respectively.

Figure 4: Historical Decomposition of Economic Growth



Notes: This figure shows both the smoothed common trend,  $\mu_t^A$ , and its historical decomposition.

Figure 5: Historical Decomposition of Cycle Components



Notes: Panel (a) through (c) show the historical decomposition of the business cycle components of real GDP, consumption and investment. Panel (d) and (e) show the historical decomposition of inflation and labor supply.