General Equilibrium Effects of the Minimum Wage

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Abstract

We examine the impact of a large minimum wage increase on macroeconomic outcomes by developing a search-matching model with heterogeneous households and an endogenously determined unemployment rate. Our model uncovers a structural mechanism underlying the employment effects of minimum wage increases, which empirical studies do not reveal. The model is calibrated to replicate the U.S. economy, enabling simulations of the minimum wage increases. We find negative non-linear effects of a substantial minimum wage increase on macroeconomic outcomes, including employment. In the $15 minimum wage scenario, output falls by approximately 3.8% relative to the baseline, while the unemployment rate rises to 6.8% from 5% in the baseline. At the individual level, the minimum wage increase enhances the welfare of high-productivity households.

Keywords: Minimum Wage, Search and Matching, Heterogeneous Agents, Mean Field Game
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1 Introduction

Since 2009, the federal minimum wage in the United States has remained at $7.25. Although the *Raise the Wage Act of 2021* attempted to raise the minimum wage to $15, legislative progress in Congress remains at a standstill. Consequently, states and localities have implemented their own policies. For example, states with more progressive labor policies, such as California and New York, have enacted substantial increases, whereas more than 20 states continue to adhere to the federal standard.[1]

The impact of minimum wage increases on employment has long been debated. Estimates exploiting natural experiments have been the standard approach since Card and Krueger [1994]. Empirical studies examining the employment effects of minimum wage increases have produced mixed results. However, surveys by Neumark and Wascher [2007] and Neumark and Shirley [2022] reveal a majority of studies suggesting a negative effect of the minimum wage on employment. Meanwhile, recent influential empirical works report no negative employment effects (e.g., Cengiz et al. [2019]; Dustmann et al. [2022]; Engbom and Moser [2022]).

However, these empirical findings based on natural experiments are unlikely to reveal the structural mechanisms underlying the employment effects of minimum wage increases. Moreover, most actual minimum wage increases remain modest at typically between $0.25 and $0.50. This limits the assessment of the effects of more substantial increases currently under discussion.

To address these issues, we develop a macroeconomic model with the following three features:

1. Heterogeneous households with individual productivity, whose steady states follow a log-normal distribution.

2. Each household has one of three employment statuses (employed, unemployment, or out of the labor force (out-of-labor)), and chooses to enter or leave the labor force based on its own productivity and asset holdings.[2]

3. A search-matching model is adopted at the contact point between firms and households. Firms pay the matching costs at the time of hiring.

[1] Wiltshire et al. [2023] reported no decrease in employment even when the minimum wage was raised to $15, although the analysis was limited to California and New York.

[2] To identify the state of out-of-labor, we assume that households in this state can gain utility from home production.

To appropriately evaluate the unemployment risk, the model assumes an incomplete market framework, following Huggett [1993] and Aiyagari [1994].[3] Households' idiosyncratic productivity, with which unemployment risk is associated, follows a diffusion process to reproduce the wage distribution, as in Alonso-Ortiz and Rogerson [2010]. The model also allows for welfare evaluation according to individual wage levels (i.e., productivity). Furthermore, for simplicity, most macroeconomic models analyzing the impact of minimum wages assume a family that shares the unemployment risk. By comparison, we employ a non-family structure. Individual risks, which are not offset or shared, add up to an aggregate shock to the economy. Thus, our model generates a priori larger negative welfare effects compared to those with complete market settings.

In addition, our model contains channels that can generate positive employment effects, as observed in existing studies. Due to the presence of hiring adjustment costs (Gertler and Trigari [2009]), the gap between the values of employment and unemployment widens as the duration of job search increases. Lower productivity households with lower job-finding rates experience larger endogenous wage markdowns than those in a frictionless economy. This is because wages are determined through Nash bargaining based on the value of employment.[4] This reflects the existence of monopsony and implies the possibility that a minimum wage increase induces out-of-labor households to enter the labor market.

Our main contribution is the proposal of a structural mechanism for the employment effects of minimum wage increases using a dynamic heterogeneous agent model. Specifically, we assess the impact of a minimum wage increase on the wage distribution through changes in employment status and conduct welfare evaluations. For example, our model endogenously replicates *excess* and *missing jobs* around the modified minimum wage threshold empirically observed by Cengiz et al. [2019]. Then, through a model simulation, we show the possible consequences of a large-scale increase in the minimum wage. In the simulation analyses, we evaluate both the long-term effects (changes in the steady state) and macroeconomic outcomes during the transition process.

This study has three main findings. First, large-scale minimum wage increases lead to significant macroeconomic outcomes, including negative employment effects. Specifically, in the $15 minimum wage scenario, output decreases by approximately 3.8% compared to

[3]We adopt a search and matching model extended following Flinn [2006], Krusell et al. [2010], and Gertler and Trigari [2009].

[4]Bagger et al. [2014] used Danish microdata, specifically matching employer-employee data, to conduct a structural estimation of an equilibrium job search model. The authors showed that job-finding rates are positively correlated with individual productivity.

the baseline, while the unemployment rate rises from 5.0% to 6.8%. Households with no employment prospects move to the out-of-labor status, causing the labor force participation rate to drop from 76.9% to 72.3%. Compared with previous studies, our larger observed effect arises because the transitions from employment to unemployment and out-of-labor among low-productivity households occur from the left tail of the wage distribution. Further, the impact increases non-linearly with the size of the minimum wage hike.

Second, the minimum wage increase improves the welfare of high-productivity households by evaluating the impact of the minimum wage increase on individual households. This is attributed to a reduction in the number of employers and general equilibrium effect, which enhances the scarcity of labor supply. Meanwhile, households with the highest wage increases (as wage markdowns are eliminated) do not experience an increase in welfare relative to the wage increase. This is because the minimum wage increase increases their unemployment risk.

Finally, the dynamic simulation shows that a temporary real minimum wage increase causes the unemployment rate to overshoot relative to the long-term equilibrium. Specifically, on the transition path immediately after a minimum wage increase, the net job loss is relatively small because excess jobs are created to partially offset missing jobs. This suggests that empirically observing a significant employment reduction immediately after a minimum wage increase may be difficult.

Among studies that examine the effects of minimum wage increases using theoretical models, Flinn [2006] is the most relevant to our study. The novelty of our study lies in incorporating a diffusion process for individual productivity into Flinn [2006]'s three-state employment model. This allows us to analyze the effects of minimum wage increases under the risk of unemployment in a heterogeneous agent model. Our approach is similar to that of Krusell et al. [2010], who incorporated idiosyncratic employment shocks into the Diamond-Mortensen-Pissarides model. While their study focuses on unemployment insurance, we examine the effects of minimum wage increases using a model in which unemployment risk depends on households' idiosyncratic productivity.

Several studies indicate that firms' monopsony power is a possible source of the positive employment effects of minimum wage increases. Theoretically, Stigler [1946] demonstrated that a minimum wage increase may increase employment under monopsony. Empirically, Staiger et al. [2010] estimated the degree of monopsony using a natural experiment based on institutional changes in the public sector. Meanwhile, Azar et al. [2024] used online job posting data and showed that higher market concentration strengthens the employment effects of minimum wage increases. Manning [2021] surveyed the increasing monopsony power of firms in recent years. Monopsony power has also been incorporated into macroeconomic models by Berger et al. [2025] and Hurst et al. [2022], who assumed upward-sloping labor supply curves due to the imperfect substitution of labor. While our study incorporates monopsony through search frictions (employment adjustment costs) and Nash bargaining in wage determination, it differs in that firms operate under perfect competition.

Regarding the unemployment risk associated with minimum wage increases, Berger et al. [2025] focused on efficient resource allocation and did not explicitly model unemployment, although they imposed rationing constraints on households. Hurst et al. [2022] and Drechsel-Grau [2023] introduced unemployment into their models despite assuming that households share risks within a family, which understates the unemployment risk. Our study is distinctive as it considers unemployment risk with a non-family structure. Furthermore, as we demonstrate the overshooting of unemployment on the transition path, we emphasize that the impact of a minimum wage increase on macroeconomic outcomes is related to the transition path and not just to steady-state changes. Hurst et al. [2022] and Drechsel-Grau [2023], who performed dynamic simulations, are complementary to our study.

The remainder of this study is organized as follows. In Section 2, we set up the model, while we calibrate its parameters in Section 3. Section 4 presents the simulation results, distinguishing between the steady state and transition path. In Section 5, we discuss the differences in the results compared with those of relevant studies. Finally, Section 6 presents our conclusions.

2 Model description

The model introduces heterogeneous households with log-normal productivity distributions to reproduce the wage distribution. Households transition between employment, unemployment, and out-of-labor statuses based on their productivity and asset holdings, while firms hire through a search-matching process and incur matching costs. To exploit these features, we assume that the households are in an incomplete market. Despite fluctuations in individual productivity and unemployment risk, households have no choice but to hedge risks by saving. For simplicity, we assume perfect competition on the production side. However, firms have monopsony power in a broad sense when wages are determined by Nash bargaining.

Households

Households choose the amount of consumption c and savings s to optimize their lifetime utility U. Each household has one of three employment statuses (employed e, unemployed u, or out-of-labor o). If they are employed, they receive labor income $w(z)$ based on their productivity, following a diffusion process; if they are unemployed or out-of-labor, they receive social benefits b instead of labor income. In addition, since a job search is not carried out while being out-of-labor, a portion of the leisure time can be used for home production γ. Income and unemployment risks cannot be shared because of (natural or restricted) borrowing constraints in the incomplete market. When unemployed, the household has the option of job searching or exiting the labor market, and entering out-of-labor.

Specifically, each household has asset holdings a, log productivity z, and employment status h as the state variables. We assume the lifetime utility of households, asset accumulation processes, and evolution of log productivity as follows:

$$\begin{gathered} U_t = \int_0^\infty e^{-\rho t} u(c_t, \gamma(h_t)) dt, \\ da_t = [e_t(z_t, h_t) - \tau_w(e_t(z_t, h_t)) \mathbb{1}_{h=e} + r_t a_t - c_t] dt = s_t dt, \ a_t \geq \underline{a}, \\ dz_t = -\eta_z z_t dt + \sigma dB_t, \end{gathered} \tag{1}$$

where c_t is consumption, s_t is savings, and r_t is the real interest rate. $\gamma(\cdot)$ is home production, $e_t(\cdot,\cdot)$ represents earnings, and $\tau_w(\cdot)$ is the progressive income tax amount defined below.[5] B_t is a Wiener process; therefore, the log productivity process is an Ornstein-Uhlenbeck process with damping parameter η_z and volatility parameter σ. Employment status h is either employed e, unemployed u, or out-of-labor o. Depending on h, home production $\gamma(\cdot)$ and earnings $e_t(\cdot,\cdot)$ take the following values:

$$\gamma(h_t) = \begin{cases} 0 & (h_t = e, u) \\ \overline{\gamma} & (h_t = o) \end{cases},$$

and

$$e_t(z_t, h_t) = \begin{cases} w_t(z_t) & (h_t = e) \\ b(z_t) & (h_t = u, o) \end{cases},$$

[5] The assumption that households directly gain utility from home production is found, for example, in Greenwood and Hercowitz [1991], Benhabib et al. [1991], or Veracierto [2008].

where $w_t(z_t)$ and $b(z_t)$ are the wage and social security benefits received by a household with log productivity z_t, respectively.[6] We specify the functional forms of the utility function $u(\cdot)$, progressive income tax $\tau_w(\cdot)$, and social security benefits $b(\cdot)$ as follows:

$$u(c,\gamma) = \frac{(c+\gamma)^{1-\theta}-1}{1-\theta},$$
$$\tau_w(w) = w - (1-\overline{\tau}_w)(w/\mathrm{E}[w])^{1-\tau}\mathrm{E}[w],$$
$$b(z) = [1 + \mathbb{1}_{w(z)\geq w_b \mathrm{Q}_{0.5}[w]}(w_b\mathrm{Q}_{0.5}[w]/w(z)-1)]w(z)\overline{b},$$

where $\mathrm{E}[w]$ and $\mathrm{Q}_{0.5}[w]$ are the mean and median wages, respectively.[7] The wage rate $w_t(z_t)$ is determined to maximize the Nash product of the household-firm matching surplus, as described below.

Let $g_t(a,z,e)$ be the density of each state of households. By definition, $\iint[g_t(a_i,z_j,e) + g_t(a_i,z_j,u) + g_t(a_i,z_j,o)]didj \equiv 1$, where we set $a_i = i$ and $z_j = j$. The employment process is determined by each household's log productivity, z_t. The ratios of employment, unemployment, and out-of-labor in the households with $z = j$—$g_{jt} = \int[g_t(a_i,z_j,e)+g_t(a_i,z_j,u)+g_t(a_i,z_j,o)]di$—are defined by $n_{jt} = g_{jt}^{-1}\int g_t(a_i,z_j,e)di$, $u_{jt} = g_{jt}^{-1}\int g_t(a_i,z_j,u)di$, and $o_{jt} \equiv 1 - n_{jt} - u_{jt}$, respectively.

Using the job-finding rate for households ζ_{jt} and a constant job separation rate q, the Hamilton-Jacobi-Bellman (HJB) equations for each household status can be expressed as follows:

$$\rho v_t^e(a_t,z_t) = \max_c[u(c_t,0) + \mathcal{A}v_t^e(a_t,z_t) + \partial_t v_t^e(a_t,z_t)] + q[v_t^u(a_t,z_t) - v_t^e(a_t,z_t)], \quad (2)$$

$$\begin{aligned}\rho v_t^u(a_t,z_t) = \max_c&[u(c_t,0) + \mathcal{A}v_t^u(a_t,z_t) + \partial_t v_t^u(a_t,z_t)] \\ &+ \mathbb{1}_{v_t^u\geq v_t^o}\zeta_t(z_t)[v_t^e(a_t,z_t) - v_t^u(a_t,z_t)] + \frac{1}{\epsilon}\mathbb{1}_{v_t^u<v_t^o}[v_t^o(a_t,z_t) - v_t^u(a_t,z_t)],\end{aligned} \quad (3)$$

$$\rho v_t^o(a_t,z_t) = \max_c[u(c_t,\overline{\gamma}) + \mathcal{A}v_t^o(a_t,z_t) + \partial_t v_t^o(a_t,z_t)] + \frac{1}{\epsilon}\mathbb{1}_{v_t^u\geq v_t^o}[v_t^u(a_t,z_t) - v_t^o(a_t,z_t)], \quad (4)$$

where $\mathcal{A} = -\eta_z z\frac{\partial}{\partial z} + \frac{1}{2}\sigma^2\frac{\partial^2}{\partial z^2} + s\frac{\partial}{\partial a}$. $\mathbb{1}_{v_t^u\geq v_t^o}$ is an indicator function that returns 1 if the value of unemployment exceeds that of out-of-labor, implying that households choose a state of

[6]Home production corresponds to the opportunity cost of search effort. More precisely, although an out-of-labor household receives social security benefits equal to unemployment benefits to keep monotonicity in the model, they should be considered as a part of home production (in addition to that in the utility function).

[7]The functional form of the progressive income amount follows that of Feldstein [1969], which is widely used in studies of taxation in heterogeneous agent models, such as Heathcote et al. [2017]. The functional form of social security benefits implies a decreasing replacement rate, which is commonly applied to unemployment benefits in the U.S. states. The adopted parameter values are described in Section 3.

greater value from unemployment and out-of-labor.[8] ϵ is the reciprocal of the time taken for the state transition between unemployment and out-of-labor. The first order condition (FOC) with respect to c_t is $\partial_c u = \partial_a v_t$.

Once the HJB equation is obtained, the corresponding Kolmogorov forward (KF) equation determines the transition of households' density $g_t(a, z, e)$ (Achdou et al. [2022]). We define aggregate effective labor as follows:

$$L_t = \left[\int \left(e^{z_j} n_{jt} \right)^{\frac{\varepsilon - 1}{\varepsilon}} g_j dj \right]^{\frac{\varepsilon}{\varepsilon - 1}},$$

where ε represents the elasticity of substitution of effective labor. The aggregate capital stock is given by the following:

$$K_t = \sum_h \iint a_i g_t(a_i, z_j, h) di dj.$$

Firms and hiring decision

Firms produce final goods under perfect competition using the labor and capital supplied by households. Because wages are determined by Nash bargaining with households, they are less than the marginal output of labor; that is, firms have monopsony power in a broad sense. Due to employment adjustment costs, the difference between the values of employment and unemployment increases when a household takes longer to find a job. Here, the wage markdown also increases. Meanwhile, households rent the capital stock at a rental price equal to its marginal productivity.

Specifically, we assume that firms are homogeneous with a constant returns to scale production function $F(K, L) = K^\alpha L^{1-\alpha}$ and the productivity distribution of workers employed at time t is identical across firms. Let the firm's hiring rate be $x_{jt} = \frac{\zeta_{jt} u_{jt}}{n_{jt}}$ and adjustment cost of employment be $\phi(x_{jt}) n_{jt}$ (Gertler and Trigari [2009]).[9] At this hiring stage, the wages w_{jt} for each productivity level, which are also paid to the incumbent workers, are given for the representative firm. Then, the firm's value of the filled job with $z = j$, $f_{jt} = f_t(n_{jt}, z_j)$,

[8] As an auxiliary explanation, we show a discretized version of the HJB equation in the Appendix.

[9] Although the model assumes the existence of a continuum number of productivity types in the model, when applied to reality, we assume that the representative firm can target more specific productivity types than, say, two or three (e.g., high school, college, or master's graduates).

satisfies the following HJB equation:

$$\rho \int f_{jt} g_{jt} dj = F(K_t, L_t) - (r_t + \delta) K_t$$
$$+ \int \{\max_x [-w_{jt} n_{jt} - \phi(x_{jt}) n_{jt} + (x_{jt} - q) n_{jt} \partial_n f_{jt}] - \eta_z z_j \partial_z f_{jt} + \frac{1}{2} \sigma^2 \partial_z^2 f_{jt} + \partial_t f_{jt} \} g_{jt} dj,$$

where the constraint $x_{jt} \in [0,\ \min\{q+\rho,\ q+u_{jt}/n_{jt}\})$ is imposed such that f_{jt} is finite and the relationship $dn_{jt} = (x_{jt} - q) n_{jt} dt$ is applied.[10]

When g_{jt} is in a steady state, the first-order condition and envelope theorem imply that:

$$\phi'(x_{jt}) = \partial_n f_{jt}, \tag{5}$$

$$\rho \partial_n f_{jt} = \mu_{jt} - w_{jt} - \phi(x_{jt}) + (x_{jt} - q) \partial_n f_{jt} - \eta_z z_j \partial_z \partial_n f_{jt} + \frac{1}{2} \sigma^2 \partial_z^2 \partial_n f_{jt} + \partial_t \partial_n f_{jt}, \tag{6}$$

where $\mu_{jt} = (1-\alpha) \left(\frac{K_t}{L_t}\right)^{\alpha} e^{z_j} \left(\frac{e^{z_j} n_{jt}}{L_t}\right)^{-\frac{1}{\varepsilon}}$ is the marginal product of labor. We specify the functional form of employment adjustment cost as $\phi(x) = \frac{1}{2} \kappa x^2$.

Wage bargaining

Wage bargaining occurs in each period, and the resulting wages are applied to all employees (including new hires). For each productivity level $z = j$, the union of workers takes the sum of the surpluses of employed workers based on the utilitarian criterion as its objective function:[11]

$$V_t(w_{jt}) = \int (v_t^e - v_t^u) g_t(a_i, z_j, e) di.$$

Meanwhile, the surplus of filled jobs with productivity level $z = j$ for the representative firm is given by the following:

$$J_t(w_{jt}) = \mu_{jt} - w_{jt}.$$

We assume that the wage rate before the minimum wage is applied, $\tilde{w}$, is determined by maximizing the Nash product:

$$\Theta_{jt} = V_t(\tilde{w}_{jt})^{\eta} J_t(\tilde{w}_{jt})^{1-\eta}.$$

[10]The Lipschitz condition is violated if x_{jt} is greater than or equal to $q + \rho$.

[11]In general, households cannot know the wealth of others and firms cannot know the wealth of their employees. We assume that workers with the same productivity (wage) unite and bargain with the representative firm.

The representative firm makes static decisions because wages are determined in each period and are not sticky. Workers make (weakly) dynamic decisions because wages affect the expected value through savings. Then, the FOC will be:

$$\eta(\mu_{jt} - \tilde{w}_{jt}) - (1-\eta)V_t(\tilde{w}_{jt})\partial_w V_t(\tilde{w}_{jt})^{-1} = 0, \tag{7}$$

where $\partial_w V_t(w_{jt}) = \int \frac{1-\tau_w'(w_{jt})}{r} \partial_a v_t^e g_t(a_i, z_j, e) di$, which is the shadow price with income as the unit of measure and converts the difference in value to that in income. The effective wage is determined by applying the minimum wage w^* to $\tilde{w}$:

$$w_{jt} = w_t(z_j) = \max\{\tilde{w}_{jt},\ w_t^*\}. \tag{8}$$

Market clearing, etc.

The real interest rate equals the net marginal productivity of capital:

$$r_t = \alpha \left(\frac{K_t}{L_t}\right)^{\alpha-1} - \delta. \tag{9}$$

The aggregate profit of the firms is given by the following:

$$\Pi_t = K_t^\alpha L_t^{1-\alpha} - W_t - (r_t + \delta)K_t - \Phi_t,$$

with the sum of wages being $W_t = \iint w_t(z_j) g_t(a_i, z_j, e) didj$ and sum of hiring costs being $\Phi_t = \iint \phi(x_{jt}) g_t(a_i, z_j, e) didj$. We assume that the government collects corporate profits in addition to the progressive income tax and provides government consumption, which is the residual of the balanced budget and social security benefits. The government's budget constraint is then given by the following:

$$G_t = \iint \tau_w(w_t(z_j)) g_t(a_i, z_j, e) didj - \iint b(z_j)[g_t(a_i, z_j, u) + g_t(a_i, z_j, o)] didj + \Pi_t.$$

Finally, the market-clearing conditions are the following:

$$Y_t = K_t^\alpha L_t^{1-\alpha} - \Phi_t,$$
$$dK_t = (Y_t - C_t - \delta K_t - G_t)dt,$$

with aggregate consumption, $C_t = \sum_h \iint c_t(a_i, z_j, h) g_t(a_i, z_j, h) didj$.

3 Calibration

We calibrate the model parameters to replicate the U.S. economy, focusing on reproducing the labor market. First, the unit of time t is set to be monthly. In countries with relatively high job-finding and separation rates, such as the U.S., a worker's status may change more than once during a period if a longer time unit is used.

Table 1 shows the parameter values, while Table 2 lists the target moments and corresponding model outcomes. We select the target moments by referring to the long-term average of the U.S. economy. Relative risk aversion θ, capital share α, and the lower bound of asset holdings $\underline{a}$ are set to 2.0, 0.33, and 0, respectively, following the literature (e.g., Achdou et al. [2022] and Gertler et al. [2008]). The discount rate ρ is set to 0.00675 to match the capital-output ratio (K/Y) to 3.0 on an annual basis. Home production $\overline{\gamma}$ is set to 0.51 to match the labor participation rate of 0.774. The capital depreciation rate, δ, is set to 0.0042 (an annual rate of 0.051). The job separation rate q is set to 4% per month, as estimated from the Job Openings and Labor Turnover Survey. The coefficient of the employment adjustment cost κ is set to 580 to obtain a steady-state unemployment rate of 5%. The labor elasticity of substitution ε is set to 5.545, as estimated from the dynamic stochastic general equilibrium (DSGE) model by Galí et al. [2012]. The bargaining weight of workers, η, is set to 0.81 to obtain a labor income share of 0.54. We assume $\epsilon = 1.0$ for numerical stability, which implies that the transition from unemployment to out-of-labor and vice versa occurs at a rate of 1 per unit time (i.e., it takes one month to fully transition).

The persistence and variance of the idiosyncratic shock to productivity faced by households, η_z and σ_z, are set to 0.0025 and 0.945, respectively, to approximate the actual distribution of the hourly wage. The hourly wage is constructed by dividing weekly earnings by weekly hours worked using microdata from the 2023 CPS. By taking the forward difference, the Ornstein-Uhlenbeck process (1) becomes $z_{t+\Delta t} - z_t = -\eta_z z_t \Delta t + \sigma(B_{t+\Delta t} - B_t)$, which can be written as follows:

$$z_{t+\Delta t} = (1 - \eta_z \Delta t) z_t + \varepsilon_t^z,$$

where ε_t^z follows a normal distribution with mean zero and variance $(\sigma \Delta t)^2$. Thus, $\eta_z = 0.0025$ implies a persistent property of each household's productivity. The steady-state distribution of labor earnings is a log-normal distribution with mean $\exp(\sigma_z^2/2)$ and variance $\exp(\sigma_z^2)(\exp(\sigma_z^2) - 1)$ where $\sigma_z^2 = \sigma^2/2\eta_z$.

The coefficients of the progressive income tax (upper panel of Figure 1) and social benefits (lower panel of Figure 1) are determined as follows: the degree of progressivity of the labor

Table 1: Parameter values for calibration

Parameter	Description	Value	Source/Target
ρ	Discount rate	0.00675	Capital-output ratio
θ	Relative risk aversion	2.0	Achdou et al. [2022]
$\overline{\gamma}$	Home production	0.51	Labor participation rate
$\underline{a}$	Borrowing limit	0	Achdou et al. [2022]
α	Capital share	0.33	Gertler et al. [2008]
δ	Capital depreciation rate	0.00423	Data
q	Separation rate	0.0406	Data
κ	Labor adjustment cost	580	Unemployment rate
ε	Substitute elasticity for labor	5.545	Galí et al. [2012]
η	Bargaining power for workers	0.81	Labor income share
η_z	Persistence of idiosyncratic shocks	0.0025	Wage distribution
σ_z	Standard deviation of shocks	0.945	Wage distribution
ϵ	Reciprocal of transition time	1.0	See text
$\overline{\tau}_w$	Marginal tax rate for average labor income	0.1807	CBO [2019]
τ	Progressivity of labor income tax	0.181	Heathcote et al. [2017]
$\overline{b}$	Social security benefits replacement rate	0.5	Institution
w_b	Threshold for replacement rate	1.07	Institution
w^*	Minimum wage	0.299	See text

income tax rate, τ, is set to 0.181 following Heathcote et al. [2017]. The shift parameter $\overline{\tau}_w$ is set to 0.1807 to replicate the average marginal tax rate of 32.9% (sum of individual income tax, 21.9%, and payroll tax, 11.0%) of U.S. labor income in 2018, as estimated by the Congressional Budget Office (CBO [2019]). The replacement rate for unemployment benefits varies somewhat by state; still, some commonality exists as it remains constant up to a certain level of pre-unemployment earnings and decreases thereafter. Here, we set the constant replacement rate $\overline{b}$ and threshold w_b to 0.5 and 1.07, respectively, by referring to the regulations in California, where the upper limit and payment period of unemployment benefits are standard in the U.S.[12] That is, the replacement rate is 50% up to 1.07 times

[12] The benefit cap in California is $450 per week, which is about the same as the $441 per week average value across all 50 states and the District of Columbia. Besides, the duration cap of 26 weeks is close to the

Table 2: Target moments

Moment	Target	Model	Source
Capital-output ratio	36.0	36.0	SNA
Labor participation rate	0.774	0.769	CPS
Unemployment rate	0.050	0.050	CPS
Labor income share	0.540	0.541	SNA
Std of log wage	0.577	0.574	CPS

Note: Target indicates data for calibration and Model indicates the corresponding model outcomes.

the median of the wage distribution; beyond that, the level remains constant and inversely proportional to the expected wage. The minimum wage w^* is set to 0.299, which is the current federal minimum wage of $7.25 standardized by the median wage.[13]

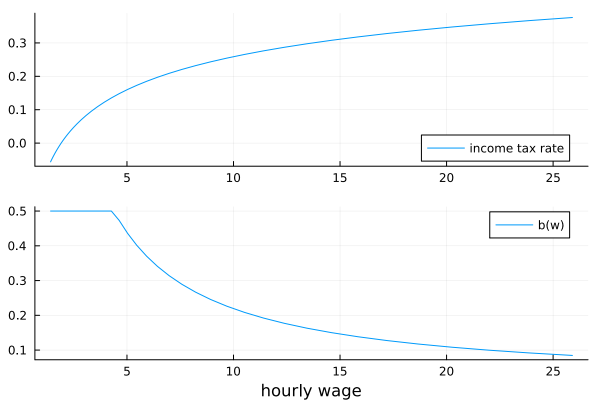

Figure 1: Progressive income tax and income replacement rates for social benefits

Note: The top panel shows income tax rates, and the bottom panel shows income replacement rates for social benefits. The functional forms are described in Section 2.

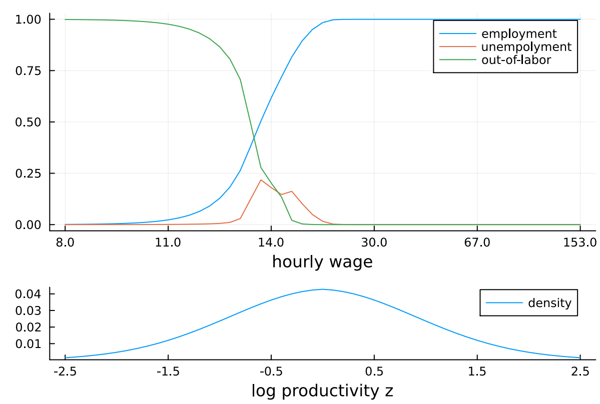

Figure 2: Distribution of employment, unemployment, and out-of-labor

Note: The top panel shows the ratio of employed, unemployed, and out-of-labor at each productivity level. The bottom panel shows the probability density of log productivity for each household in the steady state.

average of 24.4 weeks and mode of 26 weeks.

[13] Regarding the grids used in the finite difference method, we set up a lattice comprising 101 nodes from 0 to 400 at intervals of 4 for asset a and 101 nodes from -5 to 5 at intervals of 0.1 for log productivity z.

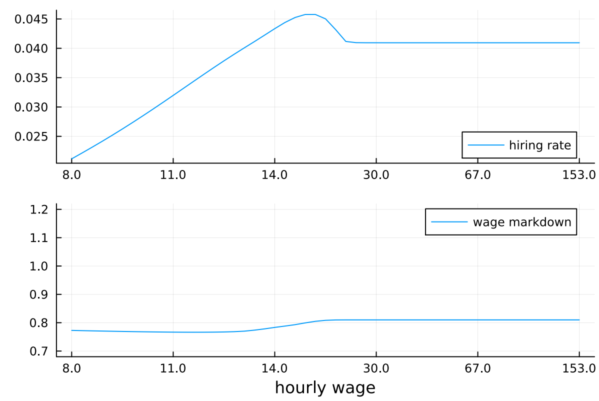

Figure 3: Hiring rates and wage markdowns

Note: The horizontal axis represents the hourly wage level, while the vertical axes represent the hiring rate and wage markdown w/μ.

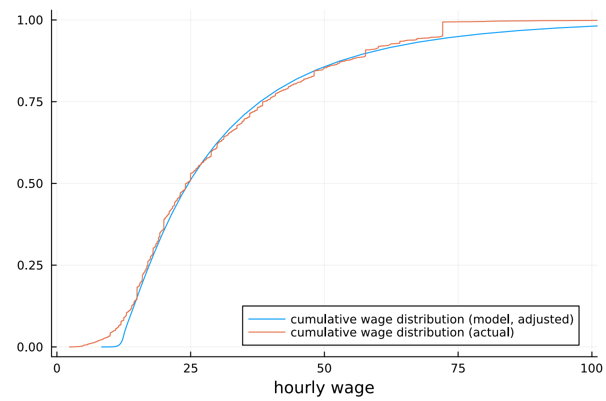

Figure 4: Wage distribution (model and actual)

Note: The wage distribution in the model (after calibration) and actual data. The latter is constructed by dividing the weekly earnings by weekly hours worked using microdata from the 2023 CPS.

4 Results

We first describe the characteristics of the steady state and dynamics with the aforementioned calibrated parameters, focusing on their distributions. We then present static simulations of raising the minimum wage, followed by a dynamic simulation.

4.1 Baseline calibration results

Figure 2 illustrates the ratios of employed, unemployed, and out-of-labor households at each productivity level in the steady state with the hourly wage. If households are highly productive, they will always be employed. However, low-productivity households prefer to be out-of-labor because the sum of home production and social security benefits exceeds the expected wages. The higher the productivity, the more profit firms earn; therefore, they pay large search costs to hire workers. Job separation occurs with exogenous probability q; however, if productivity is sufficiently high, households find jobs immediately without being unemployed. Therefore, in a high-productivity region, a firm's hiring rate is maintained at the job separation rate (see the upper panel of Figure 3).

Unemployment occurs at productivity levels slightly below the median. In a very low productivity region, firms' hiring rate x is much below the separation rate. Further, households choose out-of-labor over unemployment to obtain home production. As productivity increases, the hiring rate approaches the separation rate and households gradually become employed as they choose unemployment over being out-of-labor. The ratio of unemployment is the highest in the region where the hiring rate is slightly below the highest point. However, workers exist even in a region with no unemployment because some employees become less productive after being hired and the match is exogenously destructed with probability q.

The lower panel of Figure 3 shows the wage markdown, defined as the ratio of wages to the marginal product of labor, w/μ. In this model, wages are determined through Nash bargaining and the presence of search frictions results in a monopsonistic market where wages are set below the marginal product of labor. Even for a worker with higher productivity, the value of the wage markdown remains around 0.8 because of bargaining frictions. Moreover, workers with lower hiring rates due to lower productivity face a greater wage discount relative to their marginal product. This occurs because workers have longer job search duration, which widens the gap between the values of employment and unemployment, thereby increasing the wage markdown.

Figure 4 compares the wage distribution in the model to the actual one. The median of the model wage distribution is adjusted to match the median of the data ($24.2). Both distributions appear to be close to each other, except in the tails. However, this is unsurprising because the model reflects that the actual wage distribution can be approximated by a log-normal distribution. Notice that in the model, productivity also follows a log-normal distribution. Therefore, if the variances of both can be brought closer and because they are included in the targets of the parameter calibration, the two distributions will nearly overlap. Meanwhile, some deviations are observed in the right tail of the wage distribution.[14] Although a relatively large number of high-earnings households exist in the model, this does not affect the results because our study focuses primarily on wages and employment for those with below-median productivity.

Before moving on to our main analysis, which simulates a sizable minimum wage hike, we examine the impact of a smaller minimum wage increase, for which the empirical estimates are better studied. The magnitude of the employment effects of a minimum wage hike along the dynamic path measured in empirical studies differs from the difference between steady states. Therefore, we measure the own-wage elasticity (OWE) of our model based on a

[14]The actual wage distribution shown in Figure 4 is constructed from the CPS. Atkinson et al. [2011] noted that the survey is not reliable for the higher income bracket because of limitations in sample size, and instances of incomplete or missing responses.

Table 3: Dynamic own-wage elasticity

Year	1	2	3	4	5
Change in average wage (%)	0.108	0.180	0.189	0.156	0.093
Change in employment (%)	−0.099	−0.210	−0.234	−0.200	−0.124
OWE	−0.92	−1.17	−1.24	−1.28	−1.33

Note: The own-wage elasticity (OWE) is the ratio of the rate of change in employment to the rate of change in the average wage, measured along the transition path when the minimum wage is raised from $7.25 to $8.0.

dynamic simulation wherein the minimum wage increases from a baseline of $7.25 to $8.00 in the first period and the real minimum wage erodes thereafter at an annual rate of 2% due to inflation.

The dynamics of the OWE are presented in Table 3. It reaches a minimum of −0.92 in the first year and then gradually increases in absolute value to −1.33 in the fifth year. These values are higher than the averages reported in the literature but within the range of estimates. The survey by Neumark and Wascher [2008], for example, reported a range of −0.1 to −0.3 for OWE, while Neumark and Shirley [2022] found an average OWE of −0.148 and a median of −0.115. Dube [2019] reported a median OWE of −0.17 from 36 studies focused on the U.S. Meanwhile, some studies report relatively large OWEs in absolute values (e.g., Baskaya and Rubinstein [2012] reported −0.5, Sabia et al. [2012] reported −0.8, and Clemens and Wither [2019] reported −1.0). Notably, Clemens and Wither [2019] estimated the OWE for the series of federal minimum wage increases in 2007–2009, which were implemented during the global financial crisis. Their findings suggest that, in an environment where worker reallocation occurs less, as was the case during the global financial crisis, minimum wage hikes may lead to substantial employment declines through reduced labor demand.[15]

4.2 Minimum wage increase simulation

Next, we examine the effects of minimum wage increases from the baseline of $7.25 to $10, $12.5, and $15 (MW10, MW12.5, and MW15, respectively). All parameters other than the

[15]Note that the effect of minimum wage increases in Clemens and Wither [2019] is identified based on whether state minimum wages are bound by the federal minimum wage. As the authors also highlighted, the economic impact of the global financial crisis was more severe in states where the state's minimum wage was not bound; thus, the estimated OWE should be interpreted as a lower bound.

Table 4: Macroeconomic outcomes for each minimum wage level

Variables / Minimum wage	Baseline	MW10	MW12.5	MW15
Output ($Y_b = 100$)	100.0	99.4	98.7	96.2
Consumption ($C, Y_b = 100$)	64.2	64.1	64.1	63.8
Capital-output ratio ($K/12Y$)	3.00	3.00	2.99	2.98
Effective labor input ($L_b = 100$)	100.0	99.4	98.8	96.5
Real interest rate (%)	5.91	5.92	5.94	6.00
Mean wage (w_{mean})	34.0	34.3	34.5	35.5
Median wage (w_{median})	24.2	24.2	24.1	26.0
Labor participation rate (%)	76.9	76.5	75.7	72.3
Unemployment rate (%)	5.04	5.77	6.14	6.76
Consumption equivalence ($\Delta c, Y_b = 100$)	0.00	0.13	0.57	1.33
Social security benefits ($\Sigma b, Y_b = 100$)	3.52	3.77	4.17	5.43
Tax ($Y_b = 100$)	11.2	11.1	11.1	11.0
Profit ($Y_b = 100$)	8.0	8.0	8.0	7.9
Gov't expenditure ($G, Y_b = 100$)	15.7	15.4	14.9	13.4
Gini coefficient on wage rate	0.52	0.53	0.53	0.55
Gini coefficient on disposal income	0.40	0.40	0.40	0.40
Missing jobs ($N_b + U_b = 100$)	0.00	0.01	1.26	6.22
Excess jobs ($N_b + U_b = 100$)	0.00	0.00	0.00	2.32

Note: Y_b: baseline output, L_b: baseline effective labor, and $N_b + U_b$: baseline labor force. Disposable income includes after-tax wages, interest income, and social security benefits.

minimum wage are held at their baseline values. Table 4 summarizes the impact on output, consumption, capital-output ratio, effective labor input, real interest rate, mean wage, median wage, labor participation rate, unemployment rate, social welfare (consumption equivalence), social security benefits, tax revenue, corporate profits, government spending, Gini coefficient (wages after-tax income), missing jobs, and excess jobs (defined below). The baseline scenario represents the standard U.S. economy, for which the parameter values are calibrated to match some of its moments.

Raising the minimum wage reduces the output, consumption, capital-output ratio, effective labor input, and labor participation rate. While this increases the values of employment and unemployment by increasing wages and unemployment benefits, it also decreases hiring rates. The latter effect dominates and reduces the values of employment and unemployment, and labor participation rate. Flinn [2006] indicated a possibility that a minimum wage increase raises the value of search, thereby increasing the labor participation rate and employment. However, the author's model and empirical results, which are similar to ours, also suggested that endogenous matching probabilities reduce employment.[16]

The unemployment rate monotonically rises with a decrease in hiring rates owing to the minimum wage increase. Similarly, the mean wage monotonically increases as households with lower productivity move to the out-of-labor status. However, the median wage does not rise as much as the mean wage because the household productivity distribution has more mass in the middle than in the tails. Meanwhile, the real interest rate rises because a labor participation rate decline reduces household saving rates and investment.

Increasing the minimum wage to \$10 does not significantly affect the real economy. For example, the output decline is approximately 0.6% relative to the baseline, while the impact on consumption is negligible at −0.2%. However, the effects become significant when the minimum wage exceeds \$12.5. With a minimum wage of \$15, production declines by approximately 3.8% relative to the baseline, and consumption declines by 0.7%. The unemployment rate rises from 5.0% to 6.8%. Note that the wage distribution is similar to the log-normal distribution followed by household productivity (see Figure 4). Further, low-productivity households at the left tail of the distribution enter the unemployment or out-of-labor states first. Hence, the impact non-linearly increases.

Following Flodén [2001], we measure the impact of a minimum wage increase on social welfare by consumption equivalence, defined as Δc satisfying the following equation:

$$\frac{1}{\rho}\sum_{h}\iint u(\hat{c}_{i,jh},\gamma(h))didj = \frac{1}{\rho}\sum_{h}\iint u(\hat{c}'_{i,jh}+\Delta c,\gamma(h))didj,$$

where $\hat{c}$ on the left-hand side denotes the optimal consumption in the baseline and $\hat{c}'$ on the right-hand side denotes the optimal consumption in each simulation case (i.e., h of non-workers is also optimally chosen). A negative Δc implies that the transition to an alternative minimum wage level improves social welfare.[17] Table 4 shows that social welfare monotonically deteriorates in response to the minimum wage increase. However, even with the \$15 minimum wage, the welfare loss is 1.3% of the baseline outcome and not large because the consumption decline is moderate. Furthermore, as noted above, households that move to the out-of-labor state still gain utility from home production. Meanwhile, Berger et al.

[16]Flinn [2006] found that minimum wage increases increase labor force participation only when contract rates were treated as exogenous.

[17]The social welfare defined here is based on the Benthamite utilitarianism perspective because each household's utility is aggregated with equal weight.

[2025] found that an $11 minimum wage maximizes utilitarian welfare and increases it by 2.8%. However, the welfare gain under the efficient allocation chosen by the utilitarian planner is 30.2%. This implies that the improvement due to the minimum wage increase only accounts for one-tenth of the potential gain. Moreover, because the minimum wage increase reduces efficiency, even at the optimal level of $11, output declines by 0.1% and employment falls by 1.1% relative to the baseline. Furthermore, output and employment do not increase in response to the minimum wage increase. For instance, employment declines by more than 5% at a $15 minimum wage, consistent with our result of a 7.7% decline.

The consumption decline caused by the minimum wage hike is only minor compared with that in production because of the assumption that the income of the unemployed and out-of-labor households is compensated for by social security benefits.[18] While the social security benefits in the baseline are 3.5% of output, they increase to 5.4% under the $15 minimum wage. This difference is financed not by raising taxes, but by cutting wasteful government spending. Thus, the simulation result shows the lower limit of social welfare deterioration due to a minimum wage increase. In reality, social welfare will be worse than this.

To evaluate the impact of the minimum wage increase on wage and income inequality, we calculate the Gini coefficients. The Gini coefficient for wages monotonically increases in response to the minimum wage hike because households that worked for less than the minimum wage become unemployed or out-of-labor, relatively decreasing middle-income households. Meanwhile, when measured in terms of disposable income after accounting for social security benefits, the Gini coefficient remains unchanged. This is because raising the minimum wage increases the total amount of social security benefits during unemployment spells by assumption; this implies a significant expansion of transfers to non-workers as a trade-off.

Figure 5 illustrates the wage distribution for each minimum wage level. As in Figure 4, the horizontal and vertical axes represent the hourly wage level and cumulative employment rate, respectively. The lower bound of the distribution corresponds to the minimum wage level; raising it increases the employment bound to itself. Furthermore, a downward shift in the cumulative probability density distribution implies a total employment decline. As shown in Table 4, the employment ratio of 73% in the baseline falls to 67% at the $15 minimum wage.

[18] A lower employment rate increases unemployment spells, as shown below, and requires greater social costs, including increased social security benefits.

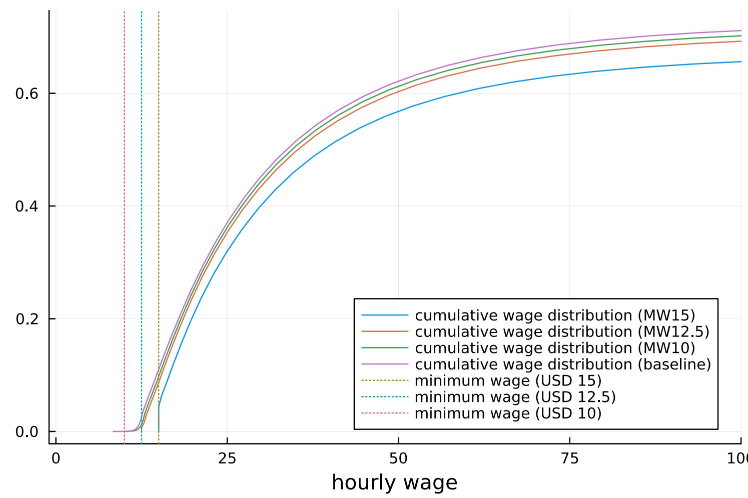

Figure 5: Wage distribution for each minimum wage level

Note: The horizontal and vertical axes represent the hourly wage level and cumulative employment rate, respectively. The dashed vertical lines show each minimum wage level.

Figure 6 shows the distribution of employment status for the baseline and $15 minimum wage, respectively, with productivity on the horizontal axis. The upper panel shows the distribution of employment and out-of-labor, where the right and left tails remain unchanged regardless of the minimum wage. Thus, as expected, the minimum wage affects the distribution only around the productivity level affected directly by it. Further, raising the minimum wage shifts the unemployment distribution to the right, as shown in the bottom panel. This implies that the minimum wage increase raises the productivity thresholds between both out-of-labor and unemployment, and unemployment and employment. The top panel of Figure 7 shows the hiring rates in the baseline and $15 minimum wage cases, whereas the bottom panel shows the wage markdowns. Raising the minimum wage reduces employment opportunities for low-productivity households. This implies the existence of a mechanism whereby wage markdowns increase when the minimum wage is binding. This reduces firm profitability, and thus, hiring rates.

Even if the wage markdown exceeds one, this does not necessarily mean that the hiring rate falls to zero. This is because the elasticity of labor substitution is finite at 5.545. Firms consider future productivity changes when determining hiring rates. However, raising the minimum wage to a binding level can be expected to significantly affect hiring and employment rates. An increase in the value of employment can lead to a transition from out-of-labor to employment via unemployment.

Figure 6: Distribution of employment, unemployment, and out-of-labor (baseline and $15 minimum wage)

Note: The horizontal and vertical axes represent log productivity and probability density for each status, respectively. The dashed vertical line shows the log productivity corresponding to the $15 minimum wage.

The left panel of Figure 8 shows the hourly wage distribution (probability density) at each minimum wage level. Cengiz et al. [2019] defined a decrease in the probability density of the hourly wage relative to the baseline as "missing jobs" and an increase as "excess jobs" (the right panel of Figure 8). We demonstrate that they arise endogenously in our model, which Cengiz et al. [2019] found empirically. They exploited 138 cases of minimum wage increases, with an average of 10.1%, to observe employment effects using a difference-in-differences (DID) approach. According to their estimates, missing jobs account for 1.8% of employment, whereas excess jobs up to +$4 from the new minimum wage are 2.1% and exceed the missing jobs. Thus, the authors concluded that a minimum wage hike increases employment.[19] The empirical fact that most excess jobs occur in the bins above the new minimum wage is consistent with our results.

In our model, raising the minimum wage reduces hiring rates, thereby reducing employment, as previously argued. For example, raising the minimum wage to $15 causes 6.22% missing jobs and 2.32% excess jobs, and the net employment decrease is 3.90% relative to the baseline labor force. Here, we define excess jobs as the sum of differences up to +$4

[19] Cengiz et al. [2019] set bins with an interval of $1 and compared the before and after treatment effects. The authors identified the sum of the differences up to +$4 from the new minimum wage as excess jobs based on statistical significance.

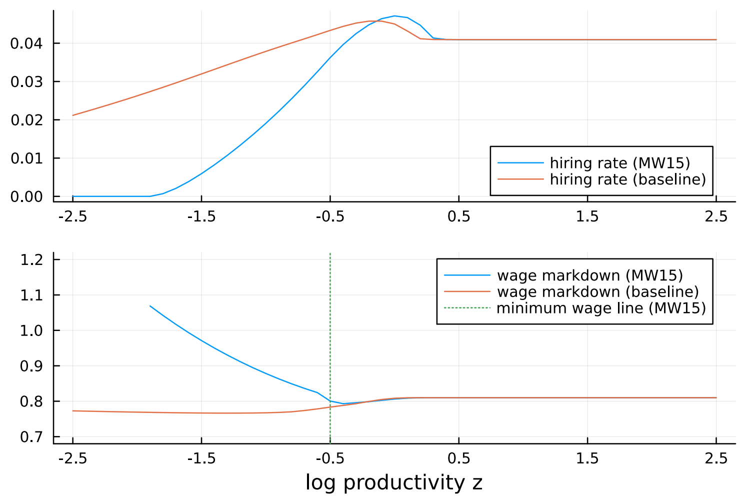

Figure 7: Hiring rates and wage markdowns (baseline and \$15 minimum wage)

Note: The horizontal axis represents log productivity. The dashed vertical line shows the log productivity corresponding to the \$15 minimum wage.

from the new minimum wage, following Cengiz et al. [2019]. The net employment losses observed here exceed their estimates. This reflects the fact that the events captured in their study are actual, modest minimum wage increases with a limited impact on missing jobs. Conversely, in our model, where the minimum wage is raised to more than twice the baseline level (to \$15), significant missing jobs emerge due to the non-linear effects on employment. Moreover, as discussed in Section 5, the exclusion of a reallocation channel from our model is also attributed to the substantial negative employment effects.

Figure 9 shows the changes in individual welfare compared with the baseline in response to the minimum wage increase. This is measured by the consumption equivalence Δc_j for each productivity j, which we define using $\tilde{\Delta} c_{ijh}$ and $\tilde{\Delta} c'_{ijh}$ satisfying the following equation:

$$\frac{1}{\rho}\sum_h \int u(\hat{c}_{ijh} + \max(\tilde{\Delta} c'_j, 0), \gamma(h))di = \frac{1}{\rho}\sum_h \int u(\hat{c}'_{ijh} + \max(\tilde{\Delta} c'_j, 0), \gamma(h))di,$$

and letting $\Delta c_j = \max(\tilde{\Delta} c_{ijh}, 0) - \max(\tilde{\Delta} c'_{ijh}, 0)$. The positivity of consumption is ensured by dividing it into two terms. In Figure 9, the left panel shows the consumption equivalence for each productivity level, whereas the right panel shows the consumption equivalence adjusted by multiplying the density of the productivity distribution such that the area represents the expected value. Social welfare decreases as the minimum wage increases, as shown in Table 4. However, the extent of welfare change depends on individual productivity.

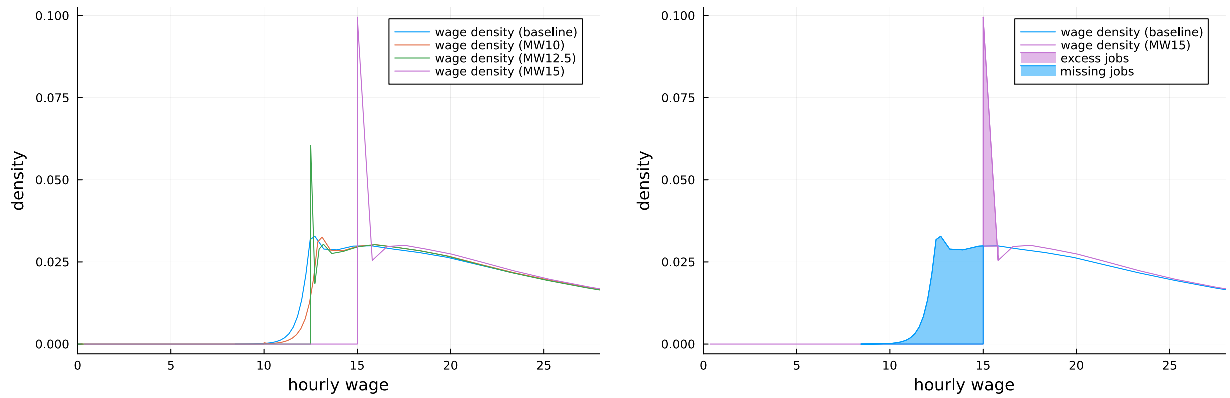

Figure 8: Wage distribution (simulation)

Note: The left panel shows the wage distribution for each minimum wage level. The right panel shows the wage distribution for the baseline and $15 minimum wage; the blue shaded area represents missing jobs, while the pink shaded area represents excess jobs in response to the increase in the minimum wage from $7.25 (baseline) to $15.

When the minimum wage is raised to $10.0 or $12.5, the welfare of workers whose productivity is close to the new minimum wage deteriorates. Meanwhile, raising the minimum wage to $15.0 improves the welfare of the fraction of workers with incomes below the new minimum wage. The reduction in the wage markdown and increase in wage levels are factors in welfare improvements but of limited magnitude. This is because the higher minimum wages reduce firms' demand for labor and increase the probability of unemployment. The capability to examine such an impact of idiosyncratic unemployment risk on expected welfare is a distinctive feature of our model, wherein workers in an incomplete market cannot share unemployment risk. Conversely, in the upper range of the productivity distribution, welfare improves uniformly since minimum wage increases reduce labor demand, resulting in fewer employees and an increased scarcity of labor supply.

Figure 10 illustrates the number of years until the employment rate exceeds 0.5, conditional on being unemployed or out-of-labor at $t = 0$. Productivity is shown on the horizontal axis (converted to the baseline wage level). Since hiring rates decline as the minimum wage increases (see the top panel of Figure 7), unemployment spells (including being out-of-labor) become prolonged, especially for individuals with lower initial productivity. As the productivity of each household varies according to Equation (1), the transition to employment is primarily driven by their productivity increase. Note that regardless of the initial productivity, the individual employment rate equals the steady-state employment rate (see Table

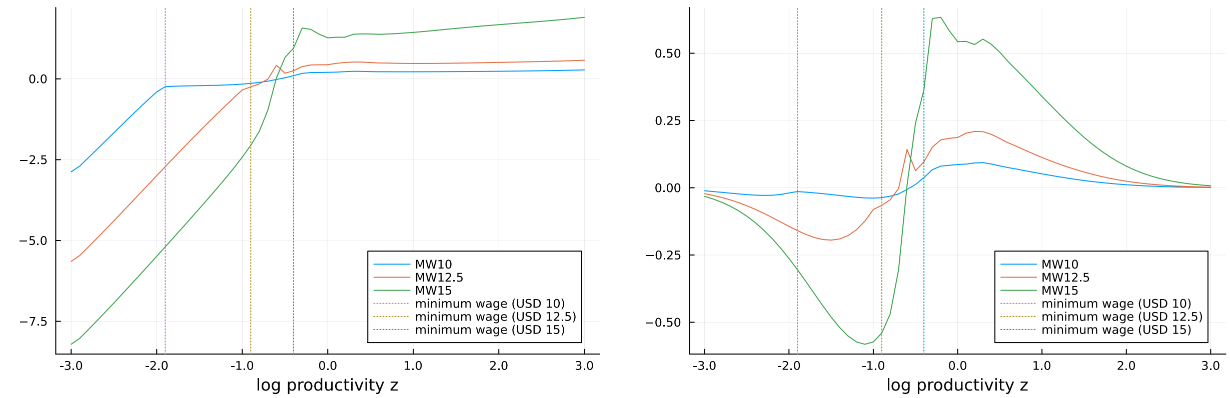

Figure 9: Consumption equivalent

Note: The figures illustrate the welfare changes relative to the baseline in response to the minimum wage increase, as measured by consumption equivalence. The left panel shows these albeit normalized by setting the baseline output to 100. The right panel shows them adjusted by multiplying the density of the productivity distribution so that the area represents the expected value.

4) after a sufficient period. Thus, raising the minimum wage extends the duration of unemployment spells in two ways. First, as already noted, it lowers hiring rates. Second, it lowers the steady-state employment rate. Prolonged unemployment spells incur substantial social costs both within the model and in the actual economy.

4.3 Dynamics analysis

We examine the transition dynamics of the minimum wage increase to analyze a more realistic scenario. The average inflation rate over the past 30 years has been approximately 2.5% in the U.S., while the monetary policy inflation target is 2%. As the minimum wage is legislated in nominal terms, its real value is eroded by inflation unless it is proportionately raised. Figure 11 shows the federal minimum wage and its real values deflated by the CPI-U. While the federal minimum wage has been raised every few years, from $0.75 in 1950 to $7.25 in 2009, the real minimum wage has hovered around $8–9 in 2023 prices.

To examine the dynamic impact on macroeconomic outcomes, we simulate a scenario in which the minimum wage increases to $15 and then left unchanged. The initial condition (year 0) is the steady state in the baseline ($7.25 minimum wage). The minimum wage is raised to $15 in year 2, followed by a 2% depreciation in real terms (upper panel of Figure 12). The terminal condition is the real minimum wage of $7.25, which is the same as the initial condition. These expectations are assumed to have perfect foresight (assuming an

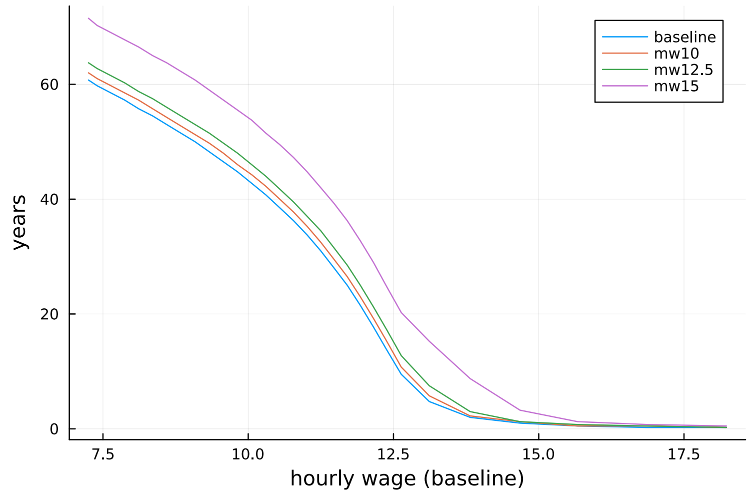

Figure 10: Unemployment duration

Note: The vertical axis represents the number of years until the employment rate exceeds 0.5, conditional on being unemployed or out-of-labor in year 0. The horizontal axis represents productivity converted to the baseline wage level.

MIT shock). That is, each economic agent makes decisions at each time point after year 1, given the paths of the exogenous variable (real minimum wage) and macro endogenous variables (wages, real interest rates, etc.) up to the steady state thereafter. Because the model is described in continuous time, the calculations are implemented after discretization with the unit of time being a month.

Figures 12–14 show the transition paths of the macroeconomic variables in response to the minimum wage increase. After it is raised to \$15 in year 2, its real value steadily depreciates by 2% inflation until it reaches \$7.25 in year 38. The mean wage peaks at \$35.3 in year 5 and then declines, returning to nearly its initial value. The mean wage increase is attributed to a composition effect resulting from the loss of low-wage employment. By the time the real minimum wage reaches the terminal condition of \$7.25, the other variables return to around their initial values.

The employment ratio falls by approximately 4 percentage points from 73% in year 5 and gradually returns to its initial level. Further, the ratio of unemployment rises from 3.9% to 6.7% in year 5, while that of out-of-labor rises from 23% to a peak of 24% in year 7. The employment dynamics are discussed in more detail below. Output drops by 1.7 percentage points by year 5, bottoms out, and then gradually returns to its initial value. This reflects that a minimum wage increase raises labor costs, thereby reducing the effective labor input

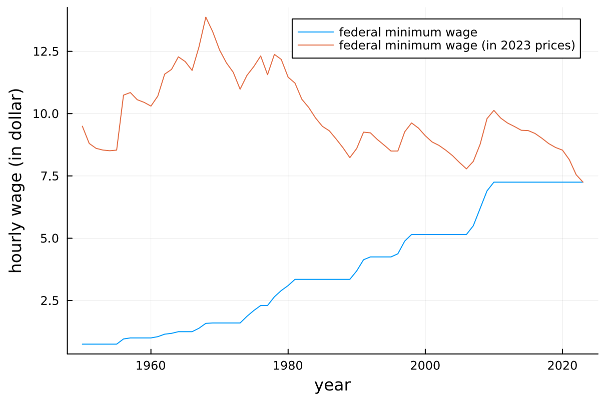

Figure 11: Nominal and real federal minimum wages

Note: The real federal minimum wage (in 2023 prices) is calculated by deflating the nominal wage by the CPI-U.

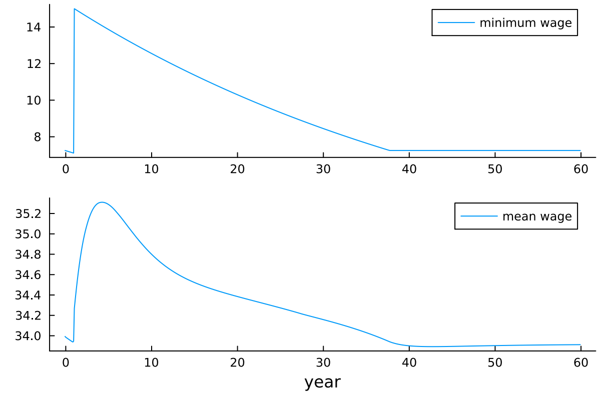

Figure 12: Minimum and mean wages

Note: The real minimum wage in the top panel is set exogenously according to the simulation scenario. The bottom panel shows the transition path of the mean wage. The vertical axes are in dollars.

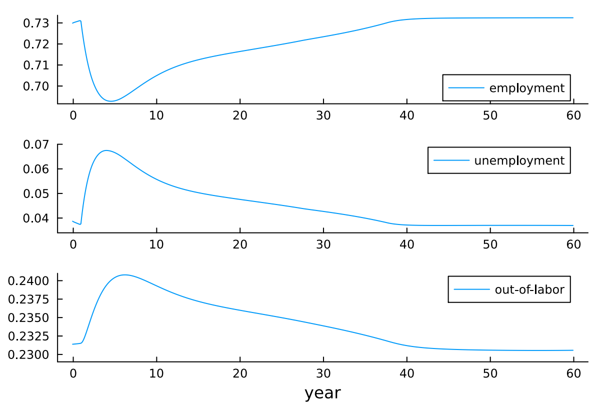

Figure 13: Employment status (transition path)

Note: Transition paths for employment (top panel), unemployment (middle panel), and out-of-labor ratios (bottom panel) following the $15 temporary minimum wage change. The vertical axes represent each state's ratio to population (= 1).

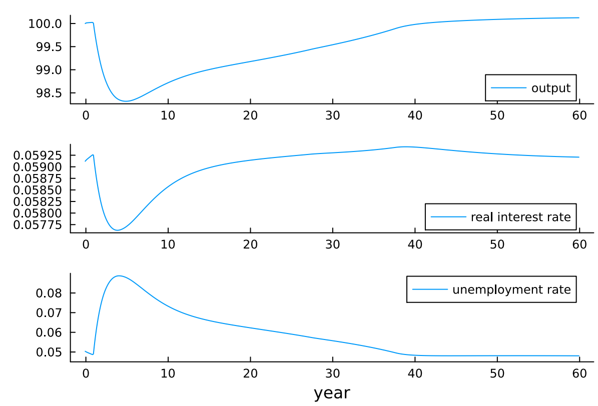

Figure 14: Output, real interest rate, and unemployment rate

Note: Transition paths for output (top panel), the real interest rate (middle panel), and the unemployment rate (bottom panel) following the $15 temporary minimum wage change.

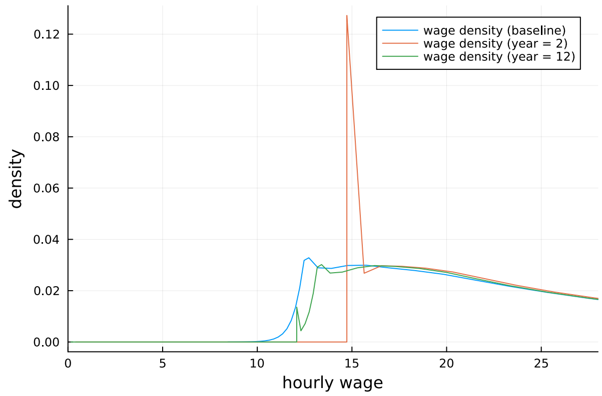

Figure 15: Wage distribution (transition path)

Note: Wage distribution at each period along the transition path following the $15 temporary minimum wage change.

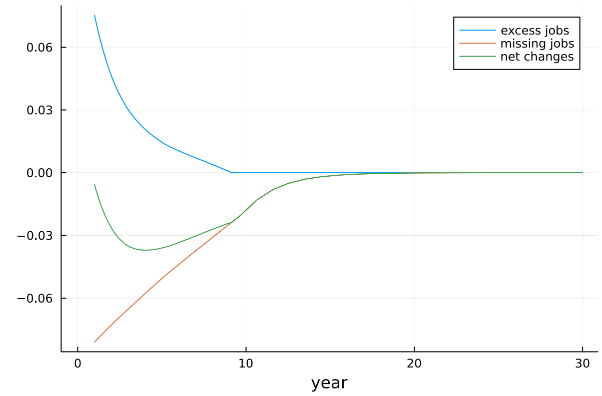

Figure 16: Excess jobs, missing jobs, and net changes

Note: Transition paths for excess jobs, missing jobs, and net employment changes following the $15 temporary minimum wage change.

and capital stock. The real interest rate also falls to 5.8% in year 4. In the steady state, with the $15 minimum wage, the real interest rate rises by 0.1 percentage points to 6.0% relative to the baseline, as shown in Table 4. However, it falls on the transition path because capital stock takes time to adjust and a temporary excess of capital stock occurs.[20]

The unemployment rate rises to 8.9% in year 5, which is well above the steady-state level of 6.8%, with the $15 minimum wage. This is because firms reserve employment anticipating a decline in the real minimum wage and households actively seek jobs anticipating future labor demand increases. Regarding the latter, the increase in the value of employment because of a higher minimum wage keeps them in the labor force rather than letting them choose the out-of-labor state.

Figure 15 illustrates the hourly wage distribution at each point along the transition path. As expected, the numbers of missing and excess jobs peak immediately after the minimum wage is raised to $15 in year 2 and are significantly larger than those in the steady state with the $15 minimum wage. Subsequently, as the real minimum wage declines, the binding wage level shifts to the left, thereby reducing both missing and excess jobs. Figure 16 shows the

[20]Hurst et al. [2022] also examined the dynamic analysis of the minimum wage increase. They included putty-clay capital in the model and found limited job losses in the short run because of the lack of capital adjustment; however, in the long run, labor is substituted for capital, thereby reducing employment for low-skilled, non-college workers.

time-series variation in missing and excess jobs. Notably, the net changes—the difference between missing and excess jobs—are relatively small in absolute value immediately after the minimum wage increase. This suggests that observing an employment decline may be difficult shortly after a minimum wage increase in natural experiments.

5 Discussion

Here, we examine how our model, its underlying assumptions, and the derived insights differ from those of relevant studies, particularly focusing on welfare outcomes. In our model, the matching between households and firms is efficient as we assume no mismatch; therefore, raising the minimum wage does not improve welfare. Conversely, other studies (e.g., Dustmann et al. [2022]) suggest that raising the minimum wage can improve economic efficiency through the reallocation effect.

A sufficient condition for the reallocation effect is that firms should have bargaining power and exert monopsony power. However, to meet this condition, the elasticity of substitution of labor supply across firms must be low if households are rational and have free movement of labor. To address this, Berger et al. [2025] and Hurst et al. [2022] introduced the family assumption; consequently, households prefer supplying labor to a wide variety of firms. This assumption seems unrealistic and underestimates the individual risks that would add up to a negative shock to the macroeconomy.

Dustmann et al. [2022] attributed the reallocation effect to the minimum wage increase shifting low-income households from small to large firms. Our model assumes perfect competition and homogeneity across firms to combine the heterogeneous agent and undirected search-matching models. In reality, if low-productivity firms retain low-income workers because of policies favoring small- and medium-sized enterprises, a higher minimum wage will improve the efficiency of matching between households and firms, and mitigate the adverse employment effect. Our model may overestimate the negative effects of minimum wage increases by not considering the reallocation effect.

The results of some empirical analyses are consistent with those results of our model. Clemens and Wither [2019], who estimated the employment effects of minimum wage hikes during the Great Recession, found an OWE of -1.0, as noted in Section 4.1. This is consistent with the OWE resulting from a modest minimum wage increase in our model. The aforementioned authors attributed the relatively large absolute estimate value to suppressed labor demand and slow productivity growth during this period. In this sense, their estimate is a lower bound and the environment is close to the assumption that reallocation hardly occurs. Thus, our simulation results should be interpreted as an upper bound on the negative employment effects of the minimum wage increase.

If firms randomly offer wages during the matching process, as in Flinn [2006] and Engbom and Moser [2022], social welfare can be improved by raising the minimum wage. However, our model deterministically determines wages through Nash bargaining between households and firms. Assuming firm heterogeneity and that matched wages are randomly determined, we may observe results similar to those of Dustmann et al. [2022] and Engbom and Moser [2022]. However, larger minimum wage increases will affect firms with moderate productivity and reduce employment, as suggested in our study. Moreover, raising the minimum wage is not the only effective solution for addressing mismatches. If matching inefficiencies exist among low-income workers, they are also more likely to exist among higher-income workers. Nevertheless, they cannot be reallocated by the minimum wage.

6 Conclusion

We construct a model with heterogeneous household productivity with an endogenously determined unemployment rate to explore the impact of a large minimum wage increase while focusing on unemployment risk. Because of the non-family structure, individual risks are neither offset nor shared, thereby adding up to an aggregate shock to the economy. We then calibrate the model parameters to replicate the U.S. economy and simulate the impact of a minimum wage increase.

Our contribution includes proposing a structural mechanism for the employment effects of minimum wage increases through the lens of a dynamic heterogeneous agent model. Our model simulations show that large-scale minimum wage increases lead to significant macroeconomic outcomes, including negative employment effects. The dynamic simulation results suggest that the net job losses immediately after a minimum wage increase are small. This finding has important implications for appropriately designing empirical analyses using natural experiments.

Our model has several limitations. Without the reallocation effect, a minimum wage increase does not improve social welfare. For example, the effect occurs if the matched wage is uncertain. To address this issue, firm heterogeneity is necessary and the impact of minimum wage increases on employment should be investigated through firm behavior. We leave this analysis to future research.

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Appendix

Bellman equation for households in discrete time

As a time discretization of the HJB equation (2)(3)(4) with $\Delta t > 0$, consider the Bellman equation (10)(11)(12):

$$v_t^e = u(c_t^*, 0)\Delta t + \beta \left\{ q\mathrm{E}_t[v_{t+\Delta t}^u - v_{t+\Delta t}^e]\Delta t + \mathrm{E}_t v_{t+\Delta t}^e \right\}, \tag{10}$$

$$v_t^u = u(c_t^*, 0)\Delta t + \beta \left\{ \max(\zeta_t \mathrm{E}_t[v_{t+\Delta t}^e - v_{t+\Delta t}^u]\Delta t, \mathrm{E}_t[v_{t+\Delta t}^o - v_{t+\Delta t}^u]) + \mathrm{E}_t v_{t+\Delta t}^u \right\}, \tag{11}$$

$$v_t^o = u(c_t^*, \overline{\gamma})\Delta t + \beta \left\{ \max(\mathrm{E}_t[v_{t+\Delta t}^u - v_{t+\Delta t}^o], 0) + \mathrm{E}_t v_{t+\Delta t}^o \right\}, \tag{12}$$

where $\beta = \frac{1}{1+\rho\Delta t}$ and c^* is the optimal consumption. Further, assume that $\mathrm{E}_t[v_{t+\Delta t}^e - v_{t+\Delta t}^u] \geq 0$ (which is valid for our model). Equation (10) is identical to the Bellman equation with the Markov jump process, for example, in Rendahl [2022], where q is the probability of a jump from employment to unemployment per unit time Δt. Equation (11) implies that a household selects the greater of the expected value gained from the possible jump, $\zeta_t \mathrm{E}_t[v_{t+\Delta t}^e - v_{t+\Delta t}^u]\Delta t$, or the difference in the expected values obtained from the transition to the out-of-labor state, $\mathrm{E}_t[v_{t+\Delta t}^o - v_{t+\Delta t}^u]$. Equation (12) also shows that a state with a high expected value is chosen. Our model is obtained by setting the limit as $\Delta t \to 0$. This operation enables the application of the finite-difference method to the HJB equation and its adjoint KF equation.[21]

Assuming that v^o in Equation (3) is an absorbing state and taking the limit as $\epsilon \to 0$, a variational inequality is derived as follows:

$$\max\left(u(c_t^*, 0) + \mathcal{A}v_t^u + \zeta_t(v_t^e - v_t^u) - \rho v_t^u,\ v_t^o - v_t^u\right) = 0.$$

However, in this case, the states of unemployment and out-of-labor cannot be mathematically separated (they can only be treated as one state). We adopt the continuous-time model (2)(3)(4) to explicitly treat the two states (in addition to the employment state).

[21]Continuous-time models, such as the one presented here, are suitable for numerical analyses that deal simultaneously with diffusion and Markov jump processes (Rendahl [2022]). An example of models with the diffusion process can be found in the online appendix of Achdou et al. [2022] or Hasumi and Takano [2025].

Online appendix (not for publication)

Brief summary of the numerical procedure

The numerical solution $\boldsymbol{v} = [\boldsymbol{v}^e \ \boldsymbol{v}^u \ \boldsymbol{v}^o]^\top$ of the HJB equation (2)(3)(4) is based on the method described in Achdou et al. [2022], as detailed below.

- Define the approximation matrix $\boldsymbol{A}_D$ of the differential operator $\mathcal{A}$ such that the advection terms are the upwind difference and second-order diffusion term is the first-order central difference.

- The jump operator is defined as follows:
$$\boldsymbol{A}_J = \begin{bmatrix} -\mathrm{diag}(q) & \mathrm{diag}(q) & O \\ \mathrm{diag}(\mathbb{1}_{\boldsymbol{v}^u \geq \boldsymbol{v}^o}\zeta) & -\mathrm{diag}(\mathbb{1}_{\boldsymbol{v}^u \geq \boldsymbol{v}^o}\zeta + \epsilon^{-1}\mathbb{1}_{\boldsymbol{v}^u < \boldsymbol{v}^o}) & \mathrm{diag}(\epsilon^{-1}\mathbb{1}_{\boldsymbol{v}^u < \boldsymbol{v}^o}) \\ O & \mathrm{diag}(\epsilon^{-1}\mathbb{1}_{\boldsymbol{v}^u \geq \boldsymbol{v}^o}) & -\mathrm{diag}(\epsilon^{-1}\mathbb{1}_{\boldsymbol{v}^u \geq \boldsymbol{v}^o}) \end{bmatrix}.$$

- We can find the steady state by solving the approximated HJB and KF equations $\rho\boldsymbol{v} = u + \boldsymbol{A}\boldsymbol{v}, \ \boldsymbol{A}^\top\boldsymbol{g} = 0$ with $\boldsymbol{A} = \boldsymbol{A}_D + \boldsymbol{A}_J$ since the conditions are $\partial_t\boldsymbol{v} = 0, \ \partial_t\boldsymbol{g} = 0$.

- x, ζ are updated from Equations (5)(6), w is updated from Equations (7)(8), and r is updated from Equation (9) in each iteration loop.

- A dynamic path is obtained as follows. After setting the initial values (in our case the initial steady state) of the state variables as the initial condition and steady state of the jump variables as the terminal condition, the transition process is obtained by solving the approximated difference equations assuming perfect foresight (also known as the MIT shock assumption).